

NATURAL CONVECTION

In Chapters 7 and 8, we considered heat transfer by *forced convection*, where a fluid was *forced* to move over a surface or in a tube by external means such as a pump or a fan. In this chapter, we consider *natural convection*, where any fluid motion occurs by natural means such as buoyancy. The fluid motion in forced convection is quite *noticeable*, since a fan or a pump can transfer enough momentum to the fluid to move it in a certain direction. The fluid motion in natural convection, however, is often not noticeable because of the low velocities involved.

The convection heat transfer coefficient is a strong function of *velocity*: the higher the velocity, the higher the convection heat transfer coefficient. The fluid velocities associated with natural convection are low, typically less than 1 m/s. Therefore, the heat transfer coefficients encountered in natural convection are usually much lower than those encountered in forced convection. Yet several types of heat transfer equipment are designed to operate under natural convection conditions instead of forced convection, because natural convection does not require the use of a fluid mover.

We start this chapter with a discussion of the physical mechanism of *natural convection* and the *Grashof number*. We then present the correlations to evaluate heat transfer by natural convection for various geometries, including finned surfaces and enclosures. Finally, we discuss simultaneous forced and natural convection.

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9-1 ■ PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 9-1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

The temperature of the air adjacent to the egg is higher, and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or “light” gas is surrounded by a high-density or “heavy” gas, and the natural laws dictate that *the light gas rise*. This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since $\rho_{\text{oil}} < \rho_{\text{vinegar}}$). This phenomenon is characterized incorrectly by the phrase “heat rises,” which is understood to mean *heated air rises*. The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current**, and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer**. Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Figure 9-2. Note that the direction of fluid motion is reversed in this case.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body

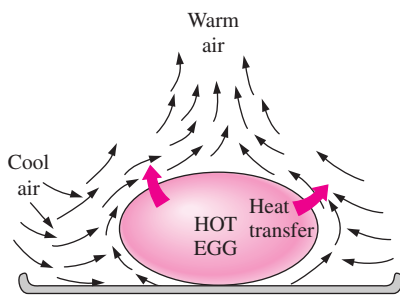


FIGURE 9-1
The cooling of a boiled egg in a cooler environment by natural convection.

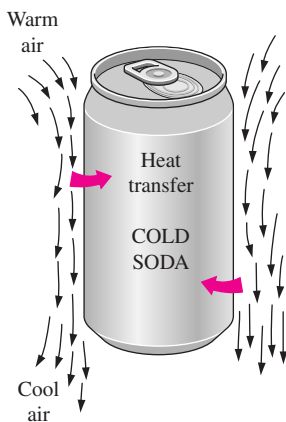
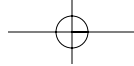


FIGURE 9-2
The warming up of a cold drink in a warmer environment by natural convection.



completely or partially immersed in it is called the **buoyancy force**. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}} g V_{\text{body}} \quad (9-1)$$

where ρ_{fluid} is the average density of the *fluid* (not the body), g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}} g V_{\text{body}} - \rho_{\text{fluid}} g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}}) g V_{\text{body}} \end{aligned} \quad (9-2)$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a “weight loss” in an amount equal to the weight of the fluid it displaces. This is known as *Archimedes’ principle*.

To have a better understanding of the buoyancy effect, consider an egg dropped into water. If the average density of the egg is greater than the density of water (a sign of freshness), the egg will settle at the bottom of the container. Otherwise, it will rise to the top. When the density of the egg equals the density of water, the egg will settle somewhere in the water while remaining completely immersed, acting like a “weightless object” in space. This occurs when the upward buoyancy force acting on the egg equals the weight of the egg, which acts downward.

The *buoyancy effect* has far-reaching implications in life. For one thing, without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by *conduction* instead of by *natural convection*. The natural convection currents encountered in the oceans, lakes, and the atmosphere owe their existence to buoyancy. Also, light boats as well as heavy warships made of steel float on water because of buoyancy (Fig. 9–3). Ships are designed on the basis of the principle that the entire weight of a ship and its contents is equal to the weight of the water that the submerged volume of the ship can contain. The “chimney effect” that induces the upward flow of hot combustion gases through a chimney is also due to the buoyancy effect, and the upward force acting on the gases in the chimney is proportional to the difference between the densities of the hot gases in the chimney and the cooler air outside. Note that there is *no gravity* in space, and thus there can be no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

In heat transfer studies, the primary variable is *temperature*, and it is desirable to express the net buoyancy force (Eq. 9-2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure*. The property that provides that information is the **volume expansion coefficient** β , defined as (Fig. 9–4)

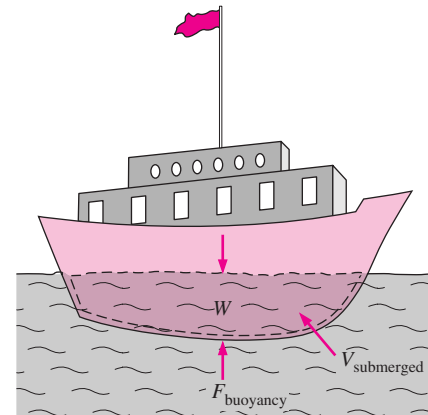
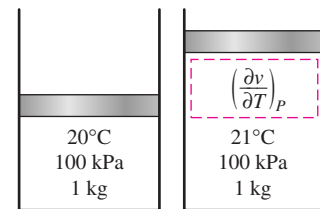
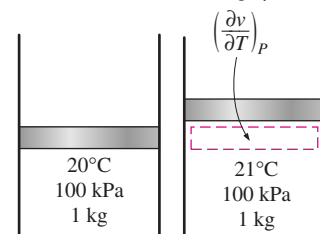


FIGURE 9-3

It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).



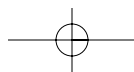
(a) A substance with a large β



(b) A substance with a small β

FIGURE 9-4

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.



$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P \quad (1/K) \quad (9-3)$$

In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T} \quad (\text{at constant } P) \quad (9-4)$$

or

$$\rho_\infty - \rho = \rho \beta (T - T_\infty) \quad (\text{at constant } P) \quad (9-5)$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the surface.

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

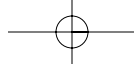
$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/K) \quad (9-6)$$

where T is the *absolute* temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure. Also note that the buoyancy force is proportional to the *density difference*, which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the *flow rate* of the fluid. The higher the flow rate, the higher the heat transfer rate. In fact, it is the very high flow rates that increase the heat transfer coefficient by orders of magnitude when forced convection is used. In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction*.

As we have discussed earlier, the buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and is proportional to this density difference and the volume occupied by the warmer fluid. It is also well known that whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a *friction force* develops at the contact surface in the direction opposite to that of the motion. This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the air flow rate driven by buoyancy is established at the point where these two effects *balance* each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection



experiments is the *Mach–Zehnder interferometer*, which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in Figure 9–5. The smooth and parallel lines in (a) indicate that the flow is *laminar*, whereas the eddies and irregularities in (b) indicate that the flow is *turbulent*. Note that the lines are closest near the surface, indicating a *higher temperature gradient*.

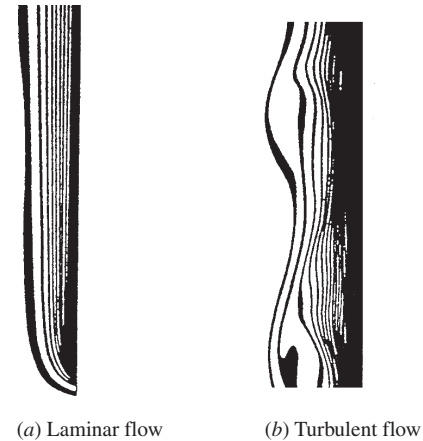
9–2 ■ EQUATION OF MOTION AND THE GRASHOF NUMBER

In this section we derive the equation of motion that governs the natural convection flow in laminar boundary layer. The conservation of mass and energy equations derived in Chapter 6 for forced convection are also applicable for natural convection, but the momentum equation needs to be modified to incorporate buoyancy.

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference $\rho - \rho_\infty$ is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. (This is known as the *Boussinesq approximation*.) We take the upward direction along the plate to be x , and the direction normal to surface to be y , as shown in Figure 9–6. Therefore, gravity acts in the $-x$ -direction. Noting that the flow is steady and two-dimensional, the x - and y -components of velocity within boundary layer are $u = u(x, y)$ and $v = v(x, y)$, respectively.

The velocity and temperature profiles for natural convection over a vertical hot plate are also shown in Figure 9–6. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as shown in the figure. In the case of *cold surfaces*, the shape of the velocity and temperature profiles remains the same but their direction is reversed.

Consider a differential volume element of height dx , length dy , and unit depth in the z -direction (normal to the paper) for analysis. The forces acting on this volume element are shown in Figure 9–7. Newton's second law of motion for this control volume can be expressed as



(a) Laminar flow

(b) Turbulent flow

FIGURE 9–5

Isotherms in natural convection over a hot plate in air.

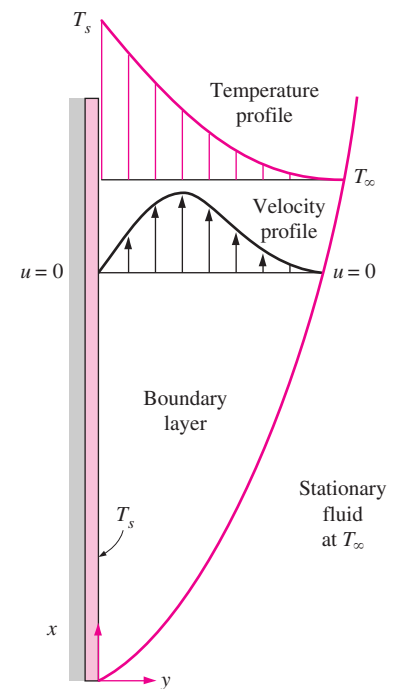
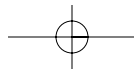


FIGURE 9–6

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞ .



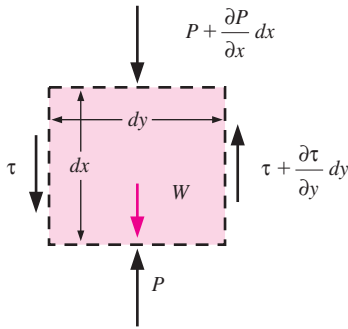
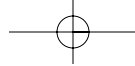


FIGURE 9-7

Forces acting on a differential control volume in the natural convection boundary layer over a vertical flat plate.

$$\delta m \cdot a_x = F_x \quad (9-7)$$

where $\delta m = \rho(dx \cdot dy \cdot 1)$ is the mass of the fluid element within the control volume. The acceleration in the x -direction is obtained by taking the total differential of $u(x, y)$, which is $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$, and dividing it by dt . We get

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \quad (9-8)$$

The forces acting on the differential volume element in the vertical direction are the pressure forces acting on the top and bottom surfaces, the shear stresses acting on the side surfaces (the normal stresses acting on the top and bottom surfaces are small and are disregarded), and the force of gravity acting on the entire volume element. Then the net surface force acting in the x -direction becomes

$$\begin{aligned} F_x &= \left(\frac{\partial \tau}{\partial y} dy \right) (dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx \right) (dy \cdot 1) - \rho g (dx \cdot dy \cdot 1) \\ &= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \right) (dx \cdot dy \cdot 1) \end{aligned} \quad (9-9)$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 9-8 and 9-9 into Eq. 9-7 and dividing by $\rho \cdot dx \cdot dy \cdot 1$ gives the *conservation of momentum* in the x -direction as

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \quad (9-10)$$

The x -momentum equation in the quiescent fluid outside the boundary layer can be obtained from the relation above as a special case by setting $u = 0$. It gives

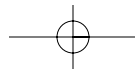
$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \quad (9-11)$$

which is simply the relation for the variation of hydrostatic pressure in a quiescent fluid with height, as expected. Also, noting that $v \ll u$ in the boundary layer and thus $\partial v/\partial x \approx \partial v/\partial y \approx 0$, and that there are no body forces (including gravity) in the y -direction, the force balance in that direction gives $\partial P/\partial y = 0$. That is, the variation of pressure in the direction normal to the surface is negligible, and for a given x the pressure in the boundary layer is equal to the pressure in the quiescent fluid. Therefore, $P = P(x) = P_\infty(x)$ and $\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$. Substituting into Eq. 9-10,

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g \quad (9-12)$$

The last term represents the net upward force per unit volume of the fluid (the difference between the buoyant force and the fluid weight). This is the force that initiates and sustains convection currents.

From Eq. 9-5, we have $\rho_\infty - \rho = \rho\beta(T - T_\infty)$. Substituting it into the last equation and dividing both sides by ρ gives the desired form of the x -momentum equation,



$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \quad (9-13)$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. Note that the momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The set of three partial differential equations (the continuity, momentum, and the energy equations) that govern natural convection flow over vertical isothermal plates can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable. But the resulting equations must still be solved numerically [Ostrach (1953), Ref. 27]. Interested reader is referred to advanced books on the topic for detailed discussions [e.g., Kays and Crawford (1993), Ref. 23].

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length L_c , all velocities by an arbitrary reference velocity \mathcal{V} (which, from the definition of Reynolds number, is taken to be $\mathcal{V} = \text{Re}_L \nu / L_c$), and temperature by a suitable temperature difference (which is taken to be $T_s - T_\infty$) as

$$x^* = \frac{x}{L_c} \quad y^* = \frac{y}{L_c} \quad u^* = \frac{u}{\mathcal{V}} \quad v^* = \frac{v}{\mathcal{V}} \quad \text{and} \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. Substituting them into the momentum equation and simplifying give

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \right] \frac{T^*}{\text{Re}_L^2} + \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (9-14)$$

The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number** Gr_L ,

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (9-15)$$

where

- g = gravitational acceleration, m/s^2
- β = coefficient of volume expansion, $1/\text{K}$ ($\beta = 1/T$ for ideal gases)
- T_s = temperature of the surface, $^\circ\text{C}$
- T_∞ = temperature of the fluid sufficiently far from the surface, $^\circ\text{C}$
- L_c = characteristic length of the geometry, m
- ν = kinematic viscosity of the fluid, m^2/s

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number*, which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number*, which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 9–8).

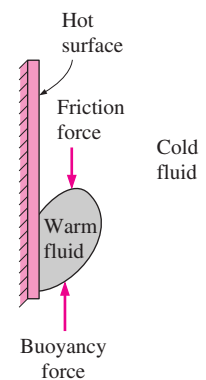


FIGURE 9–8

The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9 . Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than 10^9 .

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient Gr_L/Re_L^2 : Natural convection effects are negligible if $Gr_L/Re_L^2 \ll 1$, free convection dominates and the forced convection effects are negligible if $Gr_L/Re_L^2 \gg 1$, and both effects are significant and must be considered if $Gr_L/Re_L^2 \approx 1$.

9-3 ■ NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Some analytical solutions exist for natural convection, but such solutions lack generality since they are obtained for simple geometries under some simplifying assumptions. Therefore, with the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. Of the numerous such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and widely used.

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form (Fig. 9-9)

$$Nu = \frac{hL_c}{k} = C(Gr_L Pr)^n = C Ra_L^n \quad (9-16)$$

where Ra_L is the **Rayleigh number**, which is the product of the Grashof and Prandtl numbers:

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr \quad (9-17)$$

The values of the constants C and n depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number. The value of n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant C is normally less than 1.

Simple relations for the average Nusselt number for various geometries are given in Table 9-1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid

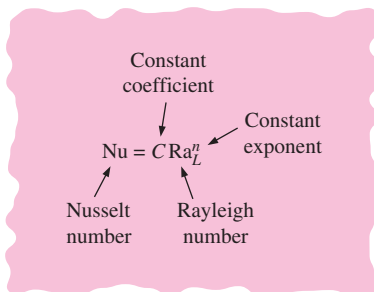


FIGURE 9-9

Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant n multiplied by another constant C , both of which are determined experimentally.

surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \quad (\text{W}) \quad (9-18)$$

where A_s is the heat transfer surface area and h is the average heat transfer coefficient on the surface.

Vertical Plates ($T_s = \text{constant}$)

For a vertical flat plate, the characteristic length is the plate height L . In Table 9-1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 9-21) recommended by Churchill and Chu (1975, Ref. 13) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < \text{Ra}_L < 10^9$.

Vertical Plates ($\dot{q}_s = \text{constant}$)

In the case of constant surface heat flux, the rate of heat transfer is known (it is simply $\dot{Q} = \dot{q}_s A_s$), but the surface temperature T_s is not. In fact, T_s increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are nearly identical [Churchill and Chu (1975), Ref. 13]. Therefore, the relations for isothermal plates can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number. Noting that $h = \dot{q}_s / (T_{L/2} - T_\infty)$, the average Nusselt number in this case can be expressed as

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \quad (9-27)$$

The midpoint temperature $T_{L/2}$ is determined by iteration so that the Nusselt numbers determined from Eqs. 9-21 and 9-27 match.

Vertical Cylinders

An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \quad (9-28)$$

When this criteria is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criteria are available in the literature [e.g., Cebeci (1974), Ref. 8].

Inclined Plates

Consider an inclined hot plate that makes an angle θ from the vertical, as shown in Figure 9-10, in a cooler environment. The net force $F = g(\rho_\infty - \rho)$ (the difference between the buoyancy and gravity) acting on a unit volume of the fluid in the boundary layer is always in the vertical direction. In the case

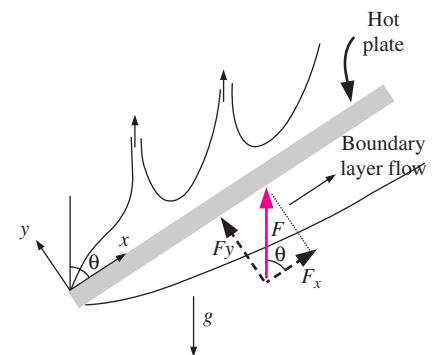


FIGURE 9-10
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

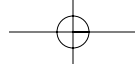
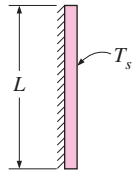
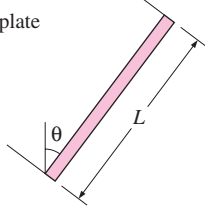

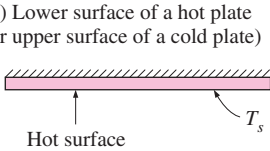
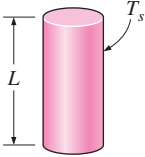
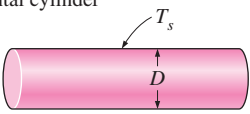
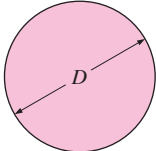
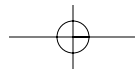
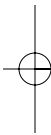
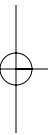


TABLE 9-1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4-10^9 10^9-10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4-10^7 10^7-10^{11} 10^5-10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)



of inclined plate, this force can be resolved into two components: $F_y = F \cos \theta$ parallel to the plate that drives the flow along the plate, and $F_y = F \sin \theta$ normal to the plate. Noting that the force that drives the motion is reduced, we expect the convection currents to be weaker, and the rate of heat transfer to be lower relative to the vertical plate case.

The experiments confirm what we suspect for the lower surface of a hot plate, but the opposite is observed on the upper surface. The reason for this curious behavior for the upper surface is that the force component F_y initiates upward motion in addition to the parallel motion along the plate, and thus the boundary layer breaks up and forms plumes, as shown in the figure. As a result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs as expected: The boundary layer on the upper surface remains intact with weaker boundary layer flow and thus lower rate of heat transfer, and the boundary layer on the lower surface breaks apart (the colder fluid falls down) and thus enhances heat transfer.

When the boundary layer remains intact (the lower surface of a hot plate or the upper surface of a cold plate), the Nusselt number can be determined from the vertical plate relations provided that g in the Rayleigh number relation is replaced by $g \cos \theta$ for $\theta < 60^\circ$. Nusselt number relations for the other two surfaces (the upper surface of a hot plate or the lower surface of a cold plate) are available in the literature [e.g., Fujii and Imura (1972), Ref. 18].

Horizontal Plates

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise. If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Figure 9–11. But if the hot surface is facing downward, the plate will block the heated fluid that tends to rise (except near the edges), impeding heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

The average Nusselt number for horizontal surfaces can be determined from the simple power-law relations given in Table 9–1. The characteristic length for horizontal surfaces is calculated from

$$L_c = \frac{A_s}{p} \quad (9-29)$$

where A_s is the surface area and p is the perimeter. Note that $L_c = a/4$ for a horizontal square surface of length a , and $D/4$ for a horizontal circular surface of diameter D .

Horizontal Cylinders and Spheres

The boundary layer over a hot horizontal cylinder start to develop at the bottom, increasing in thickness along the circumference, and forming a rising

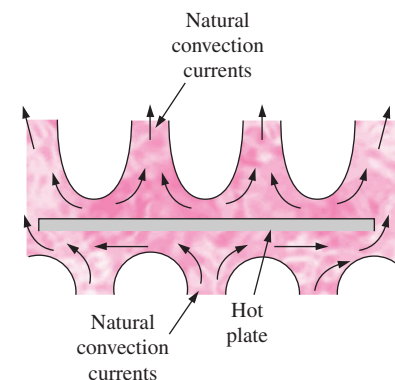


FIGURE 9–11
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

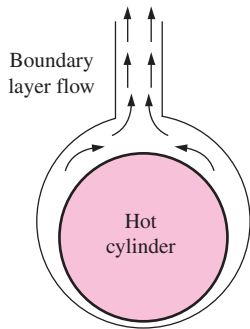
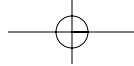


FIGURE 9-12
Natural convection flow over a horizontal hot cylinder.

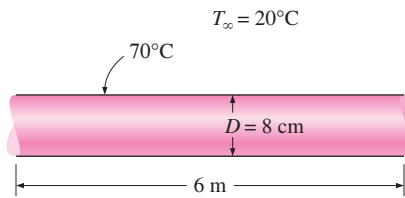


FIGURE 9-13
Schematic for Example 9-1.

plume at the top, as shown in Figure 9-12. Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar. The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

The average Nusselt number over the entire surface can be determined from Eq. 9-26 [Churchill and Chu (1975), Ref. 13] for an isothermal horizontal cylinder, and from Eq. 9-27 for an isothermal sphere [Churchill (1983), Ref. 11] both given in Table 9-1.

EXAMPLE 9-1 Heat Loss from Hot Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe shown in Figure 9-13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

SOLUTION A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45^\circ\text{C}$ and 1 atm are (Table A-15)

$$\begin{aligned} k &= 0.02699 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7241 \\ \nu &= 1.749 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_f} = \frac{1}{318 \text{ K}} \end{aligned}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08 \text{ m}$. Then the Rayleigh number becomes

$$\begin{aligned} \text{Ra}_D &= \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7241) = 1.869 \times 10^6 \end{aligned}$$

The natural convection Nusselt number in this case can be determined from Eq. 9-25 to be

$$\begin{aligned} \text{Nu} &= \left\{ 0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2 = \left\{ 0.6 + \frac{0.387(1.869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}} \right\}^2 \\ &= 17.40 \end{aligned}$$

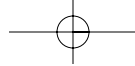
Then,

$$h = \frac{k}{D} \text{Nu} = \frac{0.02699 \text{ W/m} \cdot ^\circ\text{C}}{0.08 \text{ m}} (17.40) = 5.869 \text{ W/m} \cdot ^\circ\text{C}$$

$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869 \text{ W/m}^2 \cdot ^\circ\text{C})(1.508 \text{ m}^2)(70 - 20)^\circ\text{C} = \mathbf{443 \text{ W}}$$



Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

Discussion The pipe will lose heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity $\epsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer is determined to be (Fig. 9–14)

$$\begin{aligned} \dot{Q}_{\text{rad}} &= \epsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) \\ &= (1)(1.508 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(70 + 273 \text{ K})^4 - (20 + 273 \text{ K})^4] \\ &= 553 \text{ W} \end{aligned}$$

which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most systems cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

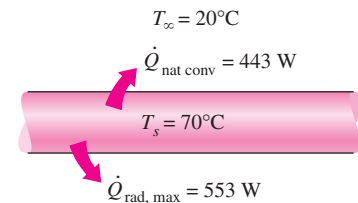


FIGURE 9–14

Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

EXAMPLE 9–2 Cooling of a Plate in Different Orientations

Consider a 0.6-m \times 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure 9–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.

SOLUTION A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60^\circ\text{C}$ and 1 atm are (Table A-15)

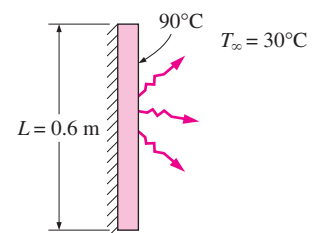
$$\begin{aligned} k &= 0.02808 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7202 \\ \nu &= 1.896 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_f} = \frac{1}{333 \text{ K}} \end{aligned}$$

Analysis (a) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6 \text{ m}$. The Rayleigh number is

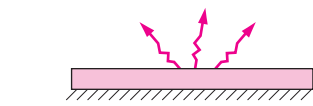
$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.722) = 7.656 \times 10^8 \end{aligned}$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

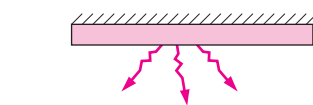
$$\begin{aligned} \text{Nu} &= \left\{ 0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}} \right\}^2 \\ &= \left\{ 0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}} \right\}^2 = 113.4 \end{aligned}$$



(a) Vertical



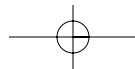
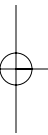
(b) Hot surface facing up



(c) Hot surface facing down

FIGURE 9–15

Schematic for Example 9–2.



Note that the simpler relation Eq. 9-19 would give $Nu = 0.59 Ra_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L} Nu = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.6 \text{ m}} (113.4) = 5.306 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{115 \text{ W}}$$

(b) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.15 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$Nu = 0.54 Ra_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.0280 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (31.76) = 5.946 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{128 \text{ W}}$$

(c) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (b). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$Nu = 0.27 Ra_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.02808 \text{ W/m} \cdot ^\circ\text{C}}{0.15 \text{ m}} (15.86) = 2.973 \text{ W/m}^2 \cdot ^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973 \text{ W/m}^2 \cdot ^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2 \text{ W}}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is “trapped” under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

Discussion The plate will lose heat to the surroundings by radiation as well as by natural convection. Assuming the surface of the plate to be black (emissivity

$\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer in this case is determined to be

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) \\ &= (1)(0.36 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(90 + 273 \text{ K})^4 - (30 + 273 \text{ K})^4] \\ &= 182 \text{ W}\end{aligned}$$

which is larger than that for natural convection heat transfer for each case. Therefore, radiation can be significant and needs to be considered in surfaces cooled by natural convection.

9-4 ■ NATURAL CONVECTION FROM FINNED SURFACES AND PCBs

Natural convection flow through a channel formed by two parallel plates as shown in Figure 9-16 is commonly encountered in practice. When the plates are hot ($T_s > T_\infty$), the ambient fluid at T_∞ enters the channel from the lower end, rises as it is heated under the effect of buoyancy, and the heated fluid leaves the channel from the upper end. The plates could be the fins of a finned heat sink, or the PCBs (printed circuit boards) of an electronic device. The plates can be approximated as being isothermal ($T_s = \text{constant}$) in the first case, and isoflux ($\dot{q}_s = \text{constant}$) in the second case.

Boundary layers start to develop at the lower ends of opposing surfaces, and eventually merge at the midplane if the plates are vertical and sufficiently long. In this case, we will have fully developed channel flow after the merger of the boundary layers, and the natural convection flow is analyzed as channel flow. But when the plates are short or the spacing is large, the boundary layers of opposing surfaces never reach each other, and the natural convection flow on a surface is not affected by the presence of the opposing surface. In that case, the problem should be analyzed as natural convection from two independent plates in a quiescent medium, using the relations given for surfaces, rather than natural convection flow through a channel.

Natural Convection Cooling of Finned Surfaces ($T_s = \text{constant}$)

Finned surfaces of various shapes, called *heat sinks*, are frequently used in the cooling of electronic devices. Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements. Natural convection is the preferred mode of heat transfer since it involves no moving parts, like the electronic components themselves. However, in the natural convection mode, the components are more likely to run at a higher temperature and thus undermine reliability. A properly selected heat sink may considerably lower the operation temperature of the components and thus reduce the risk of failure.

Natural convection from vertical finned surfaces of rectangular shape has been the subject of numerous studies, mostly experimental. Bar-Cohen and

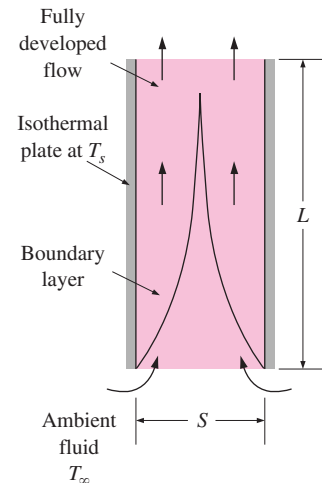


FIGURE 9-16
Natural convection flow through a channel between two isothermal vertical plates.

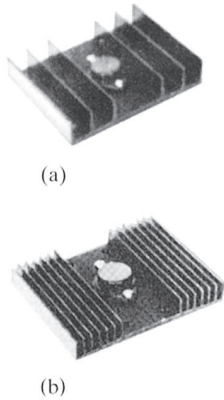
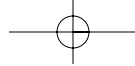


FIGURE 9-17 Heat sinks with (a) widely spaced and (b) closely packed fins (courtesy of Vemaline Products).

Rohsenow (1984, Ref. 5) have compiled the available data under various boundary conditions, and developed correlations for the Nusselt number and optimum spacing. The characteristic length for vertical parallel plates used as fins is usually taken to be the spacing between adjacent fins S , although the fin height L could also be used. The Rayleigh number is expressed as

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2} Pr \quad \text{and} \quad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr = Ra_S \frac{L^3}{S^3} \quad (9-30)$$

The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$T_s = \text{constant:} \quad Nu = \frac{hS}{k} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}} \right]^{-0.5} \quad (9-31)$$

A question that often arises in the selection of a heat sink is whether to select one with *closely packed* fins or *widely spaced* fins for a given base area (Fig. 9-17). A heat sink with closely packed fins will have greater surface area for heat transfer but a smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages. A heat sink with widely spaced fins, on the other hand, will have a higher heat transfer coefficient but a smaller surface area. Therefore, there must be an *optimum spacing* that maximizes the natural convection heat transfer from the heat sink for a given base area WL , where W and L are the width and height of the base of the heat sink, respectively, as shown in Figure 9-18. When the fins are essentially isothermal and the fin thickness t is small relative to the fin spacing S , the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

$$T_s = \text{constant:} \quad S_{opt} = 2.714 \left(\frac{S^3 L}{Ra_S} \right)^{0.25} = 2.714 \frac{L}{Ra_L^{0.25}} \quad (9-32)$$

It can be shown by combining the three equations above that when $S = S_{opt}$, the Nusselt number is a constant and its value is 1.307,

$$S = S_{opt}: \quad Nu = \frac{hS_{opt}}{k} = 1.307 \quad (9-33)$$

The rate of heat transfer by natural convection from the fins can be determined from

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \quad (9-34)$$

where $n = W/(S + t) \approx W/S$ is the number of fins on the heat sink and T_s is the surface temperature of the fins. All fluid properties are to be evaluated at the average temperature $T_{ave} = (T_s + T_\infty)/2$.

Natural Convection Cooling of Vertical PCBs ($\dot{q}_s = \text{constant}$)

Arrays of printed circuit boards used in electronic systems can often be modeled as parallel plates subjected to uniform heat flux \dot{q}_s (Fig. 9-19). The plate temperature in this case increases with height, reaching a maximum at the

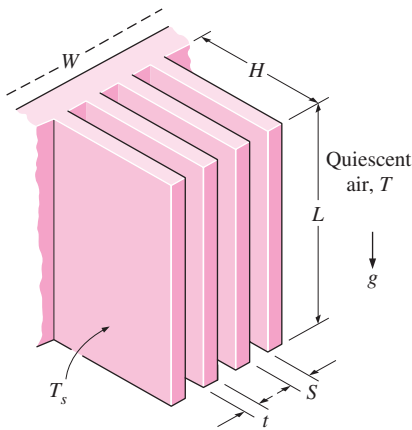
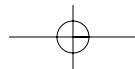
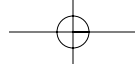


FIGURE 9-18 Various dimensions of a finned surface oriented vertically.





upper edge of the board. The modified Rayleigh number for uniform heat flux on both plates is expressed as

$$\text{Ra}_s^* = \frac{g\beta\dot{q}_s S^4}{k\nu^2} \text{Pr} \quad (9-35)$$

The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from [Bar-Cohen and Rohsenow (1984), Ref. 5]

$$\text{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\text{Ra}_s^* S/L} + \frac{2.51}{(\text{Ra}_s^* S/L)^{0.4}} \right]^{-0.5} \quad (9-36)$$

The optimum fin spacing for the case of uniform heat flux on both plates is given as

$$\dot{q}_s = \text{constant:} \quad S_{\text{opt}} = 2.12 \left(\frac{S^4 L}{\text{Ra}_s^*} \right)^{0.2} \quad (9-37)$$

The total rate of heat transfer from the plates is

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s (2nLH) \quad (9-38)$$

where $n = W/(S + t) \approx W/S$ is the number of plates. The critical surface temperature T_L occurs at the upper edge of the plates, and it can be determined from

$$\dot{q}_s = h_L (T_L - T_\infty) \quad (9-39)$$

All fluid properties are to be evaluated at the average temperature $T_{\text{ave}} = (T_L + T_\infty)/2$.

Mass Flow Rate through the Space between Plates

As we mentioned earlier, the magnitude of the natural convection heat transfer is directly related to the mass flow rate of the fluid, which is established by the dynamic balance of two opposing effects: *buoyancy* and *friction*.

The fins of a heat sink introduce both effects: *inducing extra buoyancy* as a result of the elevated temperature of the fin surfaces and *slowing down the fluid* by acting as an added obstacle on the flow path. As a result, increasing the number of fins on a heat sink can either enhance or reduce natural convection, depending on which effect is dominant. The buoyancy-driven fluid flow rate is established at the point where these two effects balance each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting fluid flow and heat transfer. Under some conditions, the increase in friction may more than offset the increase in buoyancy. This in turn will tend to reduce the flow rate and thus the heat transfer. For that reason, heat sinks with closely spaced fills are not suitable for natural convection cooling.

When the heat sink involves closely spaced fins, the narrow channels formed tend to block or “suffocate” the fluid, especially when the heat sink is long. As a result, the blocking action produced overwhelms the extra buoyancy and downgrades the heat transfer characteristics of the heat sink. Then, at a fixed power setting, the heat sink runs at a higher temperature relative to the no-shroud case. When the heat sink involves widely spaced fins, the

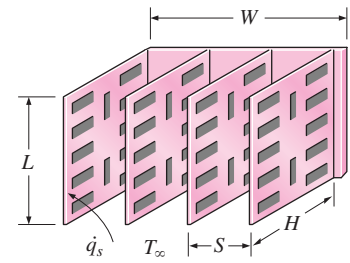
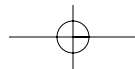
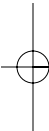


FIGURE 9-19

Arrays of vertical printed circuit boards (PCBs) cooled by natural convection.



shroud does not introduce a significant increase in resistance to flow, and the buoyancy effects dominate. As a result, heat transfer by natural convection may improve, and at a fixed power level the heat sink may run at a lower temperature.

When extended surfaces such as fins are used to enhance natural convection heat transfer between a solid and a fluid, the flow rate of the fluid in the vicinity of the solid adjusts itself to incorporate the changes in buoyancy and friction. It is obvious that this enhancement technique will work to advantage only when the increase in buoyancy is greater than the additional friction introduced. One does not need to be concerned with pressure drop or pumping power when studying natural convection since no pumps or blowers are used in this case. Therefore, an enhancement technique in natural convection is evaluated on heat transfer performance alone.

The failure rate of an electronic component increases almost exponentially with operating temperature. The cooler the electronic device operates, the more reliable it is. A rule of thumb is that the semiconductor failure rate is halved for each 10°C reduction in junction operating temperature. The desire to lower the operating temperature without having to resort to forced convection has motivated researchers to investigate enhancement techniques for natural convection. Sparrow and Prakash (Ref. 31) have demonstrated that, under certain conditions, the use of discrete plates in lieu of continuous plates of the same surface area increases heat transfer considerably. In other experimental work, using transistors as the heat source, Çengel and Zing (Ref. 9) have demonstrated that temperature recorded on the transistor case dropped by as much as 30°C when a shroud was used, as opposed to the corresponding no-shroud case.

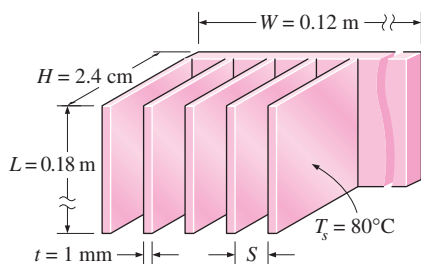


FIGURE 9-20

Schematic for Example 9-3.

EXAMPLE 9-3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Fig. 9-20). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C .

SOLUTION A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer are to be determined.

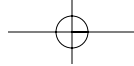
Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The atmospheric pressure at that location is 1 atm. 4 The thickness t of the fins is very small relative to the fin spacing S so that Eq. 9-32 for optimum fin spacing is applicable. 5 All fin surfaces are isothermal at base temperature.

Properties The properties of air at the film temperature of $T_f = (T_s + T_{\infty})/2 = (80 + 30)/2 = 55^{\circ}\text{C}$ and 1 atm pressure are (Table A-15)

$$k = 0.02772 \text{ W/m} \cdot ^{\circ}\text{C} \quad \text{Pr} = 0.7215$$

$$\nu = 1.846 \times 10^{-5} \text{ m}^2/\text{s} \quad \beta = 1/T_f = 1/328 \text{ K}$$

Analysis We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes



$$\begin{aligned} Ra_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr \\ &= \frac{(981 \text{ m/s}^2)[1/(328 \text{ K})](80 - 30 \text{ K})(0.18 \text{ m})^3}{(1.846 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7215) = 1.846 \times 10^7 \end{aligned}$$

The optimum fin spacing is determined from Eq. 7-32 to be

$$S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}} = 2.714 \frac{0.8 \text{ m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3} \text{ m} = \mathbf{7.45 \text{ mm}}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12 \text{ m}}{(0.00745 + 0.0001) \text{ m}} \approx 15 \text{ fins}$$

The convection coefficient for this optimum in spacing case is, from Eq. 9-33,

$$h = Nu_{opt} \frac{k}{S_{opt}} = 1.307 \frac{0.02772 \text{ W/m} \cdot \text{°C}}{0.00745 \text{ m}} = 0.2012 \text{ W/m}^2 \cdot \text{°C}$$

Then the rate of natural convection heat transfer becomes

$$\begin{aligned} \dot{Q} &= hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty) \\ &= (0.2012 \text{ W/m}^2 \cdot \text{°C})[2 \times 15(0.18 \text{ m})(0.024 \text{ m})](80 - 30) \text{°C} = \mathbf{1.30 \text{ W}} \end{aligned}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 1.30 W.

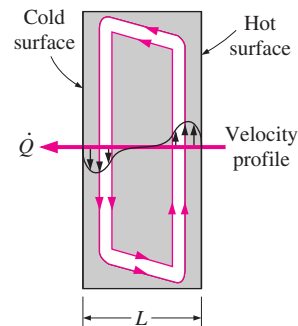


FIGURE 9-21

Convective currents in a vertical rectangular enclosure.

9-5 ■ NATURAL CONVECTION INSIDE ENCLOSURES

A considerable portion of heat loss from a typical residence occurs through the windows. We certainly would insulate the windows, if we could, in order to conserve energy. The problem is finding an insulating material that is transparent. An examination of the thermal conductivities of the insulating materials reveals that *air* is a *better insulator* than most common insulating materials. Besides, it is transparent. Therefore, it makes sense to insulate the windows with a layer of air. Of course, we need to use another sheet of glass to trap the air. The result is an *enclosure*, which is known as a *double-pane window* in this case. Other examples of enclosures include wall cavities, solar collectors, and cryogenic chambers involving concentric cylinders or spheres.

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotary motion within the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figures 9-21 and 9-22.

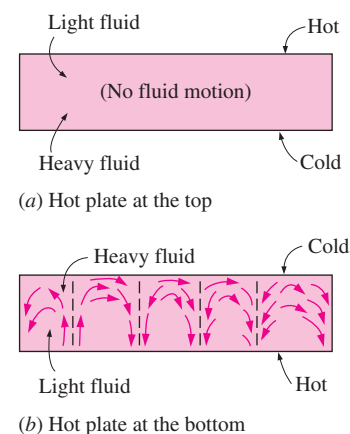
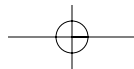


FIGURE 9-22

Convective currents in a horizontal enclosure with (a) hot plate at the top and (b) hot plate at the bottom.



The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Figure 9–22. When the hotter plate is at the *top*, no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by *pure conduction*, and we will have $Nu = 1$. When the hotter plate is at the *bottom*, the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by *pure conduction* and $Nu = 1$. When $Ra > 1708$, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells*. For $Ra > 3 \times 10^5$, the cells break down and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} Pr \quad (9-40)$$

where the characteristic length L_c is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature $T_{ave} = (T_1 + T_2)/2$.

Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = kNuA_s \frac{T_1 - T_2}{L_c} \quad (9-41)$$

since $h = kNu/L$. The rate of steady heat conduction across a layer of thickness L_c , area A_s , and thermal conductivity k is expressed as

$$\dot{Q}_{cond} = kA_s \frac{T_1 - T_2}{L_c} \quad (9-42)$$

where T_1 and T_2 are the temperatures on the two sides of the layer. A comparison of this relation with Eq. 9-41 reveals that the convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure provided that the thermal conductivity k is replaced by kNu . That is, *the fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents*. Therefore, the quantity kNu is called the **effective thermal conductivity** of the enclosure. That is,

$$k_{eff} = kNu \quad (9-43)$$

Note that for the special case of $Nu = 1$, the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid. This is expected since this case corresponds to pure conduction (Fig. 9–23).

Natural convection heat transfer in enclosed spaces has been the subject of many experimental and numerical studies, and numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of

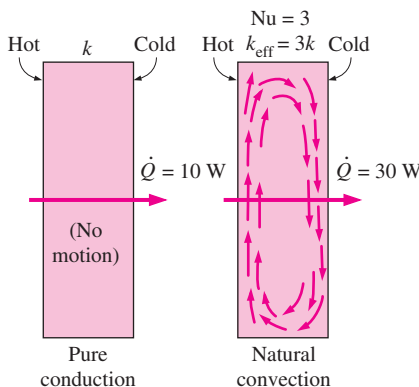


FIGURE 9–23

A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction*.

$Nu = CRa_L^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios. The relations that are more comprehensive are naturally more complex. Next we present some widely used relations for various types of enclosures.

Horizontal Rectangular Enclosures

We need no Nusselt number relations for the case of the hotter plate being at the top, since there will be no convection currents in this case and heat transfer will be downward by conduction ($Nu = 1$). When the hotter plate is at the bottom, however, significant convection currents set in for $Ra_L > 1708$, and the rate of heat transfer increases (Fig. 9–24).

For horizontal enclosures that contain air, Jakob (1949, Ref. 22) recommends the following simple correlations

$$Nu = 0.195Ra_L^{1/4} \quad 10^4 < Ra_L < 4 \times 10^5 \quad (9-44)$$

$$Nu = 0.068Ra_L^{1/3} \quad 4 \times 10^5 < Ra_L < 10^7 \quad (9-45)$$

These relations can also be used for other gases with $0.5 < Pr < 2$. Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$Nu = 0.069Ra_L^{1/3} Pr^{0.074} \quad 3 \times 10^5 < Ra_L < 7 \times 10^9 \quad (9-46)$$

Based on experiments with air, Hollands et al (1976, Ref. 19) recommend this correlation for horizontal enclosures,

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra_L} \right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1 \right]^+ \quad Ra_L < 10^8 \quad (9-47)$$

The notation $[]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for $Ra_L < 10^5$, and thus it can also be used for water.

Inclined Rectangular Enclosures

Air spaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the **aspect ratio** H/L as well as the tilt angle θ from the horizontal (Fig. 9–25).

For large aspect ratios ($H/L \geq 12$), this equation [Hollands et al., 1976, Ref. 19] correlates experimental data extremely well for tilt angles up to 70° ,

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra_L \cos \theta} \right]^+ \left(1 - \frac{1708(\sin 1.80)^{1.6}}{Ra_L \cos \theta} \right) + \left[\frac{(Ra_L \cos \theta)^{1/3}}{18} - 1 \right]^+ \quad (9-48)$$

for $Ra_L < 10^5$, $0 < \theta < 70^\circ$, and $H/L \geq 12$. Again any quantity in $[]^+$ should be set equal to zero if it is negative. This is to ensure that $Nu = 1$ for $Ra_L \cos \theta < 1708$. Note that this relation reduces to Eq. 9-47 for horizontal enclosures for $\theta = 0^\circ$, as expected.

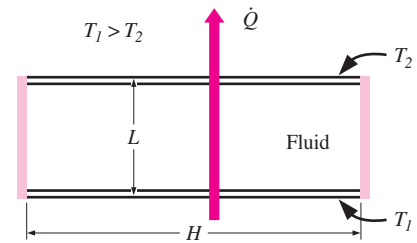


FIGURE 9–24

A horizontal rectangular enclosure with isothermal surfaces.

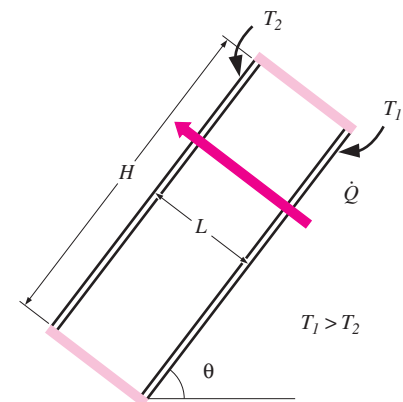


FIGURE 9–25

An inclined rectangular enclosure with isothermal surfaces.

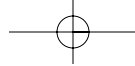


TABLE 9-2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
> 12	70°

For enclosures with smaller aspect ratios ($H/L < 12$), the next correlation can be used provided that the tilt angle is less than the critical value θ_{cr} listed in Table 9-2 [Catton (1978), Ref. 7]

$$Nu = Nu_{\theta=0^\circ} \left(\frac{Nu_{\theta=90^\circ}}{Nu_{\theta=0^\circ}} \right)^{\theta/\theta_{cr}} (\sin \theta)^{\theta/(4\theta_{cr})} \quad 0^\circ < \theta < \theta_{cr} \quad (9-49)$$

For tilt angles greater than the critical value ($\theta_{cr} < \theta < 90^\circ$), the Nusselt number can be obtained by multiplying the Nusselt number for a vertical enclosure by $(\sin \theta)^{1/4}$ [Ayyaswamy and Catton (1973), Ref. 3],

$$Nu = Nu_{\theta=90^\circ} (\sin \theta)^{1/4} \quad \theta_{cr} < \theta < 90^\circ, \text{ any } H/L \quad (9-50)$$

For enclosures tilted more than 90° , the recommended relation is [Arnold et al., (1974), Ref. 2]

$$Nu = 1 + (Nu_{\theta=90^\circ} - 1) \sin \theta \quad 90^\circ < \theta < 180^\circ, \text{ any } H/L \quad (9-51)$$

More recent but more complex correlations are also available in the literature [e.g., and ElSherbiny et al. (1982), Ref. 17].

Vertical Rectangular Enclosures

For vertical enclosures (Fig. 9-26), Catton (1978, Ref. 7) recommends these two correlations due to Berkovsky and Polevikov (1977, Ref. 6),

$$Nu = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.29} \quad \begin{array}{l} 1 < H/L < 2 \\ \text{any Prandtl number} \\ Ra_L Pr / (0.2 + Pr) > 10^3 \end{array} \quad (9-52)$$

$$Nu = 0.22 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.28} \left(\frac{H}{L} \right)^{-1/4} \quad \begin{array}{l} 2 < H/L < 10 \\ \text{any Prandtl number} \\ Ra_L < 10^{10} \end{array} \quad (9-53)$$

For vertical enclosures with larger aspect ratios, the following correlations can be used [MacGregor and Emery (1969), Ref. 26]

$$Nu = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L} \right)^{-0.3} \quad \begin{array}{l} 10 < H/L < 40 \\ 1 < Pr < 2 \times 10^4 \\ 10^4 < Ra_L < 10^7 \end{array} \quad (9-54)$$

$$Nu = 0.46 Ra_L^{1/3} \quad \begin{array}{l} 1 < H/L < 40 \\ 1 < Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{array} \quad (9-55)$$

Again all fluid properties are to be evaluated at the average temperature $(T_1 + T_2)/2$.

Concentric Cylinders

Consider two long concentric horizontal cylinders maintained at uniform but different temperatures of T_i and T_o , as shown in Figure 9-27. The diameters of the inner and outer cylinders are D_i and D_o , respectively, and the characteristic length is the spacing between the cylinders, $L_c = (D_o - D_i)/2$. The rate of heat transfer through the annular space by the natural convection unit is expressed as

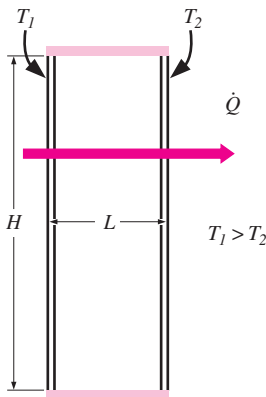


FIGURE 9-26
A vertical rectangular enclosure with isothermal surfaces.

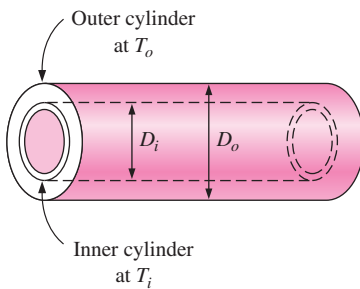
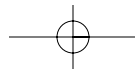


FIGURE 9-27
Two concentric horizontal isothermal cylinders.



$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o) \quad (\text{W/m}) \quad (9-56)$$

The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{k} = 0.386 \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4} \quad (9-57)$$

where the geometric factor for concentric cylinders F_{cyl} is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3 (D_i^{-3/5} + D_o^{-3/5})^5} \quad (9-58)$$

The k_{eff} relation in Eq. 9-57 is applicable for $0.70 \leq \text{Pr} \leq 6000$ and $10^2 \leq F_{\text{cyl}} \text{Ra}_L \leq 10^7$. For $F_{\text{cyl}} \text{Ra}_L < 100$, natural convection currents are negligible and thus $k_{\text{eff}} = k$. Note that k_{eff} cannot be less than k , and thus we should set $k_{\text{eff}} = k$ if $k_{\text{eff}}/k < 1$. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

Concentric Spheres

For concentric isothermal spheres, the rate of heat transfer through the gap between the spheres by natural convection is expressed as (Fig. 9–28)

$$\dot{Q} = k_{\text{eff}} \pi \left(\frac{D_i D_o}{L_c} \right) (T_i - T_o) \quad (\text{W}) \quad (9-59)$$

where $L_c = (D_o - D_i)/2$ is the characteristic length. The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{k} = 0.74 \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4} \quad (9-60)$$

where the geometric factor for concentric spheres F_{sph} is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \quad (9-61)$$

The k_{eff} relation in Eq. 9-60 is applicable for $0.70 \leq \text{Pr} \leq 4200$ and $10^2 \leq F_{\text{sph}} \text{Ra}_L \leq 10^4$. If $k_{\text{eff}}/k < 1$, we should set $k_{\text{eff}} = k$.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the air space of a double pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \quad (9-62)$$

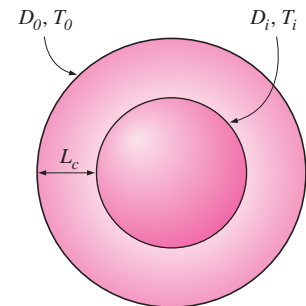


FIGURE 9–28
Two concentric isothermal spheres.

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} (both in absolute temperature unit K) is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4) \quad (\text{W}) \quad (9-63)$$

where ε is the emissivity of the surface, A_s is the surface area, and $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ is the Stefan–Boltzmann constant.

When the end effects are negligible, radiation heat transfer between two large parallel plates at absolute temperatures T_1 and T_2 is expressed as (see Chapter 12 for details)

$$\dot{Q}_{\text{rad}} = \frac{\sigma A_s (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{\text{effective}} \sigma A_s (T_1^4 - T_2^4) \quad (\text{W}) \quad (9-64)$$

where ε_1 and ε_2 are the emissivities of the plates, and $\varepsilon_{\text{effective}}$ is the *effective emissivity* defined as

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \quad (9-65)$$

The emissivity of an ordinary glass surface, for example, is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. Radiation heat transfer between concentric cylinders and spheres is discussed in Chapter 12.

Note that in some cases the temperature of the surrounding medium may be below the surface temperature ($T_\infty < T_s$), while the temperature of the surrounding surfaces is above the surface temperature ($T_{\text{surr}} > T_s$). In such cases, convection and radiation heat transfers are subtracted from each other instead of being added since they are in opposite directions. Also, for a metal surface, the radiation effect can be reduced to negligible levels by polishing the surface and thus lowering the surface emissivity to a value near zero.

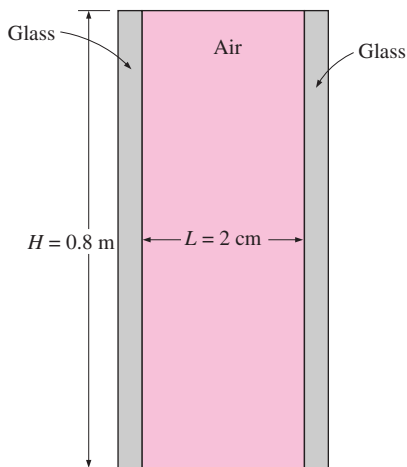


FIGURE 9–29
Schematic for Example 9–4.

EXAMPLE 9–4 Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 9–29 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

SOLUTION Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{\text{ave}} = (T_1 + T_2)/2 = (12 + 2)/2 = 7^\circ\text{C}$ and 1 atm pressure are (Table A-15)

$$\begin{aligned} k &= 0.02416 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7344 \\ \nu &= 1.399 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_{\text{ave}}} = \frac{1}{280 \text{ K}} \end{aligned}$$

Analysis We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $L = 0.02 \text{ m}$. Then the Rayleigh number becomes

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_1 - T_2)L^3}{\nu^2} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(280 \text{ K})](12 - 2 \text{ K})(0.02 \text{ m})^3}{(1.399 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7344) = 1.051 \times 10^4 \end{aligned}$$

The aspect ratio of the geometry is $H/L = 0.8/0.02 = 40$. Then the Nusselt number in this case can be determined from Eq. 9-54 to be

$$\begin{aligned} \text{Nu} &= 0.42\text{Ra}_L^{1/4} \text{Pr}^{0.012} \left(\frac{H}{L}\right)^{-0.3} \\ &= 0.42(1.051 \times 10^4)^{1/4} (0.7344)^{0.012} \left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401 \end{aligned}$$

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\begin{aligned} \dot{Q} &= hA_s(T_1 - T_2) = k\text{Nu}A_s \frac{T_1 - T_2}{L} \\ &= (0.02416 \text{ W/m} \cdot ^\circ\text{C})(1.401)(1.6 \text{ m}^2) \frac{(12 - 2)^\circ\text{C}}{0.02 \text{ m}} = \mathbf{27.1 \text{ W}} \end{aligned}$$

Therefore, heat will be lost through the window at a rate of 27.1 W.

Discussion Recall that a Nusselt number of $\text{Nu} = 1$ for an enclosure corresponds to pure conduction heat transfer through the enclosure. The air in the enclosure in this case remains still, and no natural convection currents occur in the enclosure. The Nusselt number in our case is 1.32, which indicates that heat transfer through the enclosure is 1.32 times that by pure conduction. The increase in heat transfer is due to the natural convection currents that develop in the enclosure.

EXAMPLE 9-5 Heat Transfer through a Spherical Enclosure

The two concentric spheres of diameters $D_i = 20 \text{ cm}$ and $D_o = 30 \text{ cm}$ shown in Fig. 9-30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320 \text{ K}$ and $T_o = 280 \text{ K}$, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{\text{ave}} = (T_i + T_o)/2 = (320 + 280)/2 = 300 \text{ K} = 27^\circ\text{C}$ and 1 atm pressure are (Table A-15)

$$\begin{aligned} k &= 0.02566 \text{ W/m} \cdot ^\circ\text{C} & \text{Pr} &= 0.7290 \\ \nu &= 1.580 \times 10^{-5} \text{ m}^2/\text{s} & \beta &= \frac{1}{T_{\text{ave}}} = \frac{1}{300 \text{ K}} \end{aligned}$$

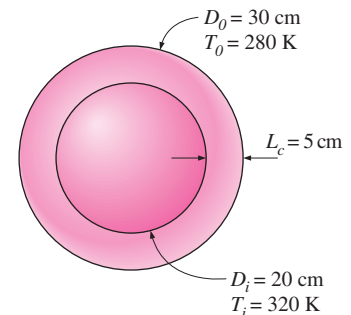


FIGURE 9-30
Schematic for Example 9-5.

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L^3}{\nu^2} \text{Pr} \\ &= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.729) = 4.776 \times 10^5 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{\text{sph}} &= \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \\ &= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4 [(0.2 \text{ m})^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229 \end{aligned}$$

$$\begin{aligned} k_{\text{eff}} &= 0.74k \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4} \\ &= 0.74(0.02566 \text{ W/m} \cdot ^\circ\text{C}) \left(\frac{0.729}{0.861 + 0.729} \right) (0.005229 \times 4.776 \times 10^5)^{1/4} \\ &= 0.1104 \text{ W/m} \cdot ^\circ\text{C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\begin{aligned} \dot{Q} &= k_{\text{eff}} \pi \left(\frac{D_i D_o}{L_c} \right) (T_i - T_o) \\ &= (0.1104 \text{ W/m} \cdot ^\circ\text{C}) \pi \left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}} \right) (320 - 280) \text{ K} = \mathbf{16.7 \text{ W}} \end{aligned}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

Discussion Note that the air in the spherical enclosure will act like a stationary fluid whose thermal conductivity is $k_{\text{eff}}/k = 0.1104/0.02566 = 4.3$ times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually very significant, and should be considered in a complete analysis.

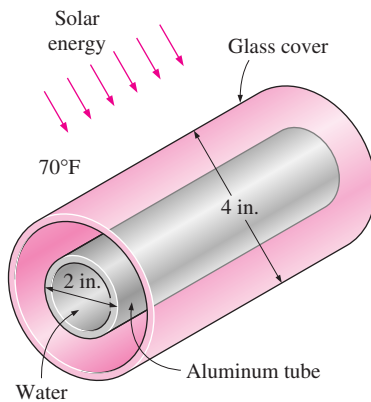


FIGURE 9-31
Schematic for Example 9-6.

EXAMPLE 9-6 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in.-diameter (Fig. 9-31). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of $(70 + 110)/2 = 90^\circ\text{F}$ (Table A-15E),

$$k = 0.01505 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \quad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753 \times 10^{-4} \text{ ft}^2/\text{s} \quad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550 \text{ K}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \quad (\text{per foot of tube})$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \quad (\text{per foot of tube})$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 100°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{D_o} = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2} \text{Pr}$$

$$= \frac{(32.2 \text{ ft/s}^2)[1/(550 \text{ R})](110 - 70 \text{ R})(4/12 \text{ ft})^3}{(1.753 \times 10^{-4} \text{ ft}^2/\text{s})^2} (0.7275) = 2.054 \times 10^6$$

$$\text{Nu} = \left\{ 0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2 = \left\{ 0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}} \right\}^2$$

$$= 17.89$$

$$h_o = \frac{k}{D_o} \text{Nu} = \frac{0.0150 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}}{4/12 \text{ ft}} (17.89) = 0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

$$\dot{Q}_o = h_o A_o (T_o - T_\infty) = (0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(1.047 \text{ ft}^2)(110 - 70)^\circ\text{F}$$

$$= 33.8 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in.} = 1/12 \text{ ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of $(106 + 200)/2 = 154^\circ\text{F} = 614 \text{ R}$. This gives

$$\begin{aligned} \text{Ra}_L &= \frac{g\beta(T_i - T_o)L_c^3}{\nu^2} \text{Pr} \\ &= \frac{(32.2 \text{ ft/s}^2)[1/614 \text{ R}](200 - 106 \text{ R})(1/12 \text{ ft})^3}{(2.117 \times 10^{-4} \text{ ft}^2/\text{s})^2} (0.7184) = 4.579 \times 10^4 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{\text{cyl}} &= \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \\ &= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466 \end{aligned}$$

$$\begin{aligned} k_{\text{eff}} &= 0.386k \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4} \\ &= 0.386(0.01653 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}) \left(\frac{0.7184}{0.861 + 0.7184} \right) (0.1466 \times 4.579 \times 10^4)^{1/4} \\ &= 0.04743 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \end{aligned}$$

Then the rate of heat transfer between the cylinders becomes

$$\begin{aligned} \dot{Q} &= \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o) \\ &= \frac{2\pi(0.04743 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F})}{\ln(4/2)} (200 - 106)^\circ\text{F} = 40.4 \text{ Btu/h} \end{aligned}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F**. Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.

Discussion Note that we have not considered heat loss by radiation in the calculations, and thus the tube temperature determined above is probably too high. This problem is considered again in Chapter 12 by accounting for the effect of radiation heat transfer.

9-6 ■ COMBINED NATURAL AND FORCED CONVECTION

The presence of a temperature gradient in a fluid in a gravity field always gives rise to natural convection currents, and thus heat transfer by natural convection. Therefore, forced convection is always accompanied by natural convection.

We mentioned earlier that the convection heat transfer coefficient, natural or forced, is a strong function of the fluid velocity. Heat transfer coefficients encountered in forced convection are typically much higher than those encountered in natural convection because of the higher fluid velocities associated with forced convection. As a result, we tend to ignore natural convection in heat transfer analyses that involve forced convection, although we recognize that natural convection always accompanies forced convection. The error involved in ignoring natural convection is negligible at high velocities but may be considerable at low velocities associated with forced convection. Therefore, it is desirable to have a criterion to assess the relative magnitude of natural convection in the presence of forced convection.

For a given fluid, it is observed that the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. This is not surprising since the convection heat transfer coefficient is a strong function of the Reynolds number Re in forced convection and the Grashof number Gr in natural convection.

A plot of the nondimensionalized heat transfer coefficient for combined natural and forced convection on a vertical plate is given in Fig. 9–32 for different fluids. We note from this figure that natural convection is negligible when $Gr/Re^2 < 0.1$, forced convection is negligible when $Gr/Re^2 > 10$, and neither is negligible when $0.1 < Gr/Re^2 < 10$. Therefore, both natural and forced convection must be considered in heat transfer calculations when the Gr and Re^2 are of the same order of magnitude (one is within a factor of 10 times the other). Note that forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

Natural convection may *help* or *hurt* forced convection heat transfer, depending on the relative directions of *buoyancy-induced* and the *forced convection* motions (Fig. 9–33):

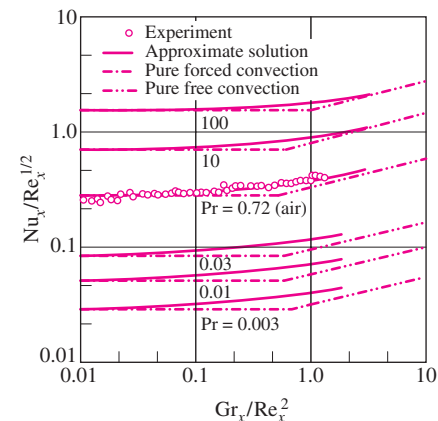


FIGURE 9–32 Variation of the local Nusselt number NU_x for combined natural and forced convection from a hot isothermal vertical plate (from Lloyd and Sparrow, Ref. 25).

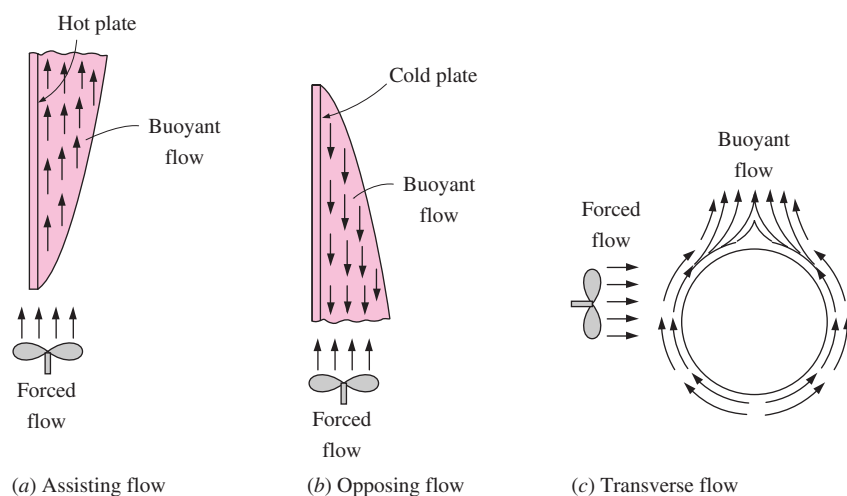


FIGURE 9–33

Natural convection can *enhance* or *inhibit* heat transfer, depending on the relative directions of *buoyancy-induced* motion and the *forced convection* motion.

1. In *assisting flow*, the buoyant motion is in the *same* direction as the forced motion. Therefore, natural convection assists forced convection and *enhances* heat transfer. An example is upward forced flow over a hot surface.
2. In *opposing flow*, the buoyant motion is in the *opposite* direction to the forced motion. Therefore, natural convection resists forced convection and *decreases* heat transfer. An example is upward forced flow over a cold surface.
3. In *transverse flow*, the buoyant motion is *perpendicular* to the forced motion. Transverse flow enhances fluid mixing and thus *enhances* heat transfer. An example is horizontal forced flow over a hot or cold cylinder or sphere.

When determining heat transfer under combined natural and forced convection conditions, it is tempting to add the contributions of natural and forced convection in assisting flows and to subtract them in opposing flows. However, the evidence indicates differently. A review of experimental data suggests a correlation of the form

$$\text{Nu}_{\text{combined}} = (\text{Nu}_{\text{forced}}^n \pm \text{Nu}_{\text{natural}}^n)^{1/n} \quad (9-41)$$

where $\text{Nu}_{\text{forced}}$ and $\text{Nu}_{\text{natural}}$ are determined from the correlations for *pure forced* and *pure natural convection*, respectively. The plus sign is for *assisting* and *transverse* flows and the minus sign is for *opposing* flows. The value of the exponent n varies between 3 and 4, depending on the geometry involved. It is observed that $n = 3$ correlates experimental data for vertical surfaces well. Larger values of n are better suited for horizontal surfaces.

A question that frequently arises in the cooling of heat-generating equipment such as electronic components is whether to use a fan (or a pump if the cooling medium is a liquid)—that is, whether to utilize *natural* or *forced* convection in the cooling of the equipment. The answer depends on the maximum allowable operating temperature. Recall that the convection heat transfer rate from a surface at temperature T_s in a medium at T_∞ is given by

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient and A_s is the surface area. Note that for a fixed value of power dissipation and surface area, h and T_s are *inversely proportional*. Therefore, the device will operate at a *higher* temperature when h is low (typical of natural convection) and at a *lower* temperature when h is high (typical of forced convection).

Natural convection is the preferred mode of heat transfer since no blowers or pumps are needed and thus all the problems associated with these, such as noise, vibration, power consumption, and malfunctioning, are avoided. Natural convection is adequate for cooling *low-power-output* devices, especially when they are attached to extended surfaces such as heat sinks. For *high-power-output* devices, however, we have no choice but to use a blower or a pump to keep the operating temperature below the maximum allowable level. For *very-high-power-output* devices, even forced convection may not be sufficient to keep the surface temperature at the desirable levels. In such cases, we may have to use *boiling* and *condensation* to take advantage of the very high heat transfer coefficients associated with phase change processes.

TOPIC OF SPECIAL INTEREST*

Heat Transfer Through Windows

Windows are *glazed apertures* in the building envelope that typically consist of single or multiple *glazing* (glass or plastic), *framing*, and *shading*. In a building envelope, windows offer the *least resistance* to heat flow. In a typical house, about *one-third* of the total heat loss in winter occurs through the windows. Also, most air infiltration occurs at the edges of the windows. The solar heat gain through the windows is responsible for much of the cooling load in summer. The net effect of a window on the heat balance of a building depends on the characteristics and orientation of the window as well as the solar and weather data. Workmanship is very important in the construction and installation of windows to provide effective sealing around the edges while allowing them to be opened and closed easily.

Despite being so undesirable from an energy conservation point of view, windows are an essential part of any building envelope since they enhance the appearance of the building, allow *daylight* and *solar heat* to come in, and allow people to view and observe outside without leaving their home. For low-rise buildings, windows also provide easy exit areas during emergencies such as fire. Important considerations in the selection of windows are *thermal comfort* and *energy conservation*. A window should have a good light transmittance while providing effective resistance to heat flow. The lighting requirements of a building can be minimized by maximizing the use of natural daylight. Heat loss in winter through the windows can be minimized by using airtight double- or triple-pane windows with spectrally selective films or coatings, and letting in as much solar radiation as possible. Heat gain and thus cooling load in summer can be minimized by using effective internal or external shading on the windows.

Even in the absence of solar radiation and air infiltration, heat transfer through the windows is more complicated than it appears to be. This is because the structure and properties of the frame are quite different than the glazing. As a result, heat transfer through the frame and the edge section of the glazing adjacent to the frame is two-dimensional. Therefore, it is customary to consider the window in three regions when analyzing heat transfer through it: (1) the *center-of-glass*, (2) the *edge-of-glass*, and (3) the *frame* regions, as shown in Figure 9–34. Then the total rate of heat transfer through the window is determined by adding the heat transfer through each region as

$$\begin{aligned}\dot{Q}_{\text{window}} &= \dot{Q}_{\text{center}} + \dot{Q}_{\text{edge}} + \dot{Q}_{\text{frame}} \\ &= U_{\text{window}} A_{\text{window}} (T_{\text{indoors}} - T_{\text{outdoors}})\end{aligned}\quad (9-67)$$

where

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}} \quad (9-68)$$

is the **U-factor** or the **overall heat transfer coefficient** of the window; A_{window} is the window area; A_{center} , A_{edge} , and A_{frame} are the areas of the

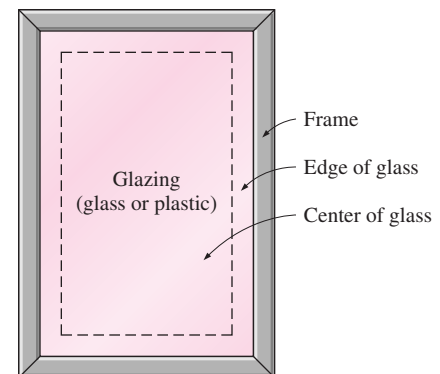


FIGURE 9–34

The three regions of a window considered in heat transfer analysis.

*This section can be skipped without a loss of continuity.

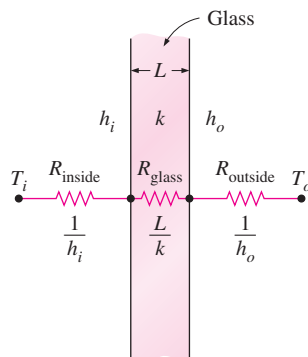


FIGURE 9-35

The thermal resistance network for heat transfer through a single glass.

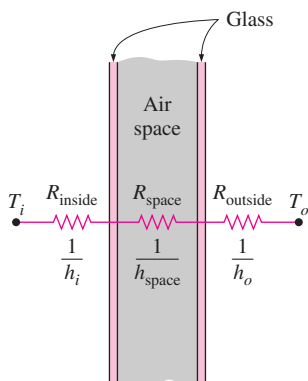


FIGURE 9-36

The thermal resistance network for heat transfer through the center section of a double-pane window (the resistances of the glasses are neglected).

center, edge, and frame sections of the window, respectively; and U_{center} , U_{edge} , and U_{frame} are the heat transfer coefficients for the center, edge, and frame sections of the window. Note that $A_{\text{window}} = A_{\text{center}} + A_{\text{edge}} + A_{\text{frame}}$, and the overall U -factor of the window is determined from the area-weighted U -factors of each region of the window. Also, the inverse of the U -factor is the R -value, which is the unit thermal resistance of the window (thermal resistance for a unit area).

Consider steady one-dimensional heat transfer through a single-pane glass of thickness L and thermal conductivity k . The thermal resistance network of this problem consists of surface resistances on the inner and outer surfaces and the conduction resistance of the glass in series, as shown in Figure 9-35, and the total resistance on a unit area basis can be expressed as

$$R_{\text{total}} = R_{\text{inside}} + R_{\text{glass}} + R_{\text{outside}} = \frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o} \quad (9-69)$$

Using common values of 3 mm for the thickness and $0.92 \text{ W/m} \cdot ^\circ\text{C}$ for the thermal conductivity of the glass and the winter design values of 8.29 and $34.0 \text{ W/m}^2 \cdot ^\circ\text{C}$ for the inner and outer surface heat transfer coefficients, the thermal resistance of the glass is determined to be

$$R_{\text{total}} = \frac{1}{8.29 \text{ W/m}^2 \cdot ^\circ\text{C}} + \frac{0.003 \text{ m}}{0.92 \text{ W/m} \cdot ^\circ\text{C}} + \frac{1}{34.0 \text{ W/m}^2 \cdot ^\circ\text{C}}$$

$$= 0.121 + 0.003 + 0.029 = 0.153 \text{ m}^2 \cdot ^\circ\text{C/W}$$

Note that the ratio of the glass resistance to the total resistance is

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{0.003 \text{ m}^2 \cdot ^\circ\text{C/W}}{0.153 \text{ m}^2 \cdot ^\circ\text{C/W}} = 2.0\%$$

That is, the glass layer itself contributes about 2 percent of the total thermal resistance of the window, which is negligible. The situation would not be much different if we used acrylic, whose thermal conductivity is $0.19 \text{ W/m} \cdot ^\circ\text{C}$, instead of glass. Therefore, we cannot reduce the heat transfer through the window effectively by simply increasing the thickness of the glass. But we can reduce it by trapping still air between two layers of glass. The result is a **double-pane window**, which has become the norm in window construction.

The thermal conductivity of air at room temperature is $k_{\text{air}} = 0.025 \text{ W/m} \cdot ^\circ\text{C}$, which is one-thirtieth that of glass. Therefore, the thermal resistance of 1-cm-thick still air is equivalent to the thermal resistance of a 30-cm-thick glass layer. Disregarding the thermal resistances of glass layers, the thermal resistance and U -factor of a double-pane window can be expressed as (Fig. 9-36)

$$\frac{1}{U_{\text{double-pane (center region)}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o} \quad (9-70)$$

where $h_{\text{space}} = h_{\text{rad, space}} + h_{\text{conv, space}}$ is the combined radiation and convection heat transfer coefficient of the space trapped between the two glass layers.

Roughly half of the heat transfer through the air space of a double-pane window is by radiation and the other half is by conduction (or convection,

if there is any air motion). Therefore, there are two ways to minimize h_{space} and thus the rate of heat transfer through a double-pane window:

1. Minimize radiation heat transfer through the air space. This can be done by reducing the emissivity of glass surfaces by coating them with low-emissivity (or “low-e” for short) material. Recall that the *effective emissivity* of two parallel plates of emissivities ε_1 and ε_2 is given by

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \quad (9-71)$$

The emissivity of an ordinary glass surface is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. But when the glass surfaces are coated with a film that has an emissivity of 0.1, the effective emissivity reduces to 0.05, which is one-fourteenth of 0.72. Then for the same surface temperatures, radiation heat transfer will also go down by a factor of 14. Even if only one of the surfaces is coated, the overall emissivity reduces to 0.1, which is the emissivity of the coating. Thus it is no surprise that about one-fourth of all windows sold for residences have a low-e coating. The heat transfer coefficient h_{space} for the air space trapped between the two vertical parallel glass layers is given in Table 9–3 for 13-mm- ($\frac{1}{2}$ -in.) and 6-mm- ($\frac{1}{4}$ -in.) thick air spaces for various effective emissivities and temperature differences.

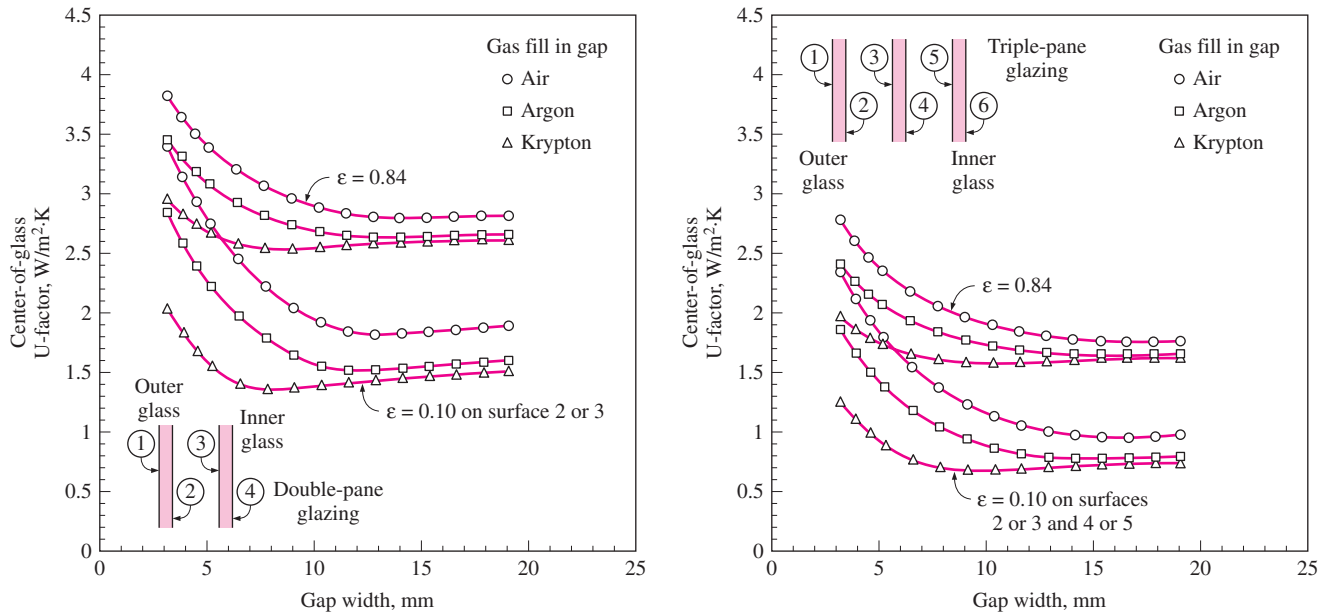
It can be shown that coating just one of the two parallel surfaces facing each other by a material of emissivity ε reduces the effective emissivity nearly to ε . Therefore, it is usually more economical to coat only one of the facing surfaces. Note from Figure 9–37 that coating one of the interior surfaces of a double-pane window with a material having an emissivity of 0.1

TABLE 9–3

The heat transfer coefficient h_{space} for the air space trapped between the two vertical parallel glass layers for 13-mm- and 6-mm-thick air spaces (from Building Materials and Structures, Report 151, U.S. Dept. of Commerce).

(a) Air space thickness = 13 mm						(b) Air space thickness = 6 mm					
		$h_{\text{space}}, \text{W/m}^2 \cdot \text{°C}^*$						$h_{\text{space}}, \text{W/m}^2 \cdot \text{°C}^*$			
$T_{\text{ave}},$ °C	$\Delta T,$ °C	$\varepsilon_{\text{effective}}$				$T_{\text{ave}},$ °C	$\Delta T,$ °C	$\varepsilon_{\text{effective}}$			
		0.72	0.4	0.2	0.1			0.72	0.4	0.2	0.1
0	5	5.3	3.8	2.9	2.4	0	5	7.2	5.7	4.8	4.3
0	15	5.3	3.8	2.9	2.4	0	50	7.2	5.7	4.8	4.3
0	30	5.5	4.0	3.1	2.6	10	5	7.7	6.0	5.0	4.5
10	5	5.7	4.1	3.0	2.5	10	50	7.7	6.1	5.0	4.5
10	15	5.7	4.1	3.1	2.5	30	5	8.8	6.8	5.5	4.9
10	30	6.0	4.3	3.3	2.7	30	50	8.8	6.8	5.5	4.9
30	5	5.7	4.6	3.4	2.7	50	5	10.0	7.5	6.0	5.2
30	15	5.7	4.7	3.4	2.8	50	50	10.0	7.5	6.0	5.2
30	30	6.0	4.9	3.6	3.0						

*Multiply by 0.176 to convert to $\text{Btu/h} \cdot \text{ft}^2 \cdot \text{°F}$.



(a) Double-pane window

(b) Triple-pane window

FIGURE 9-37

The variation of the U -factor for the center section of double- and triple-pane windows with uniform spacing between the panes (from ASHRAE *Handbook of Fundamentals*, Ref. 1, Chap. 27, Fig. 1).

reduces the rate of heat transfer through the center section of the window by half.

2. Minimize conduction heat transfer through air space. This can be done by *increasing* the distance d between the two glasses. However, this cannot be done indefinitely since increasing the spacing beyond a critical value initiates convection currents in the enclosed air space, which increases the heat transfer coefficient and thus defeats the purpose. Besides, increasing the spacing also increases the thickness of the necessary framing and the cost of the window. Experimental studies have shown that when the spacing d is less than about 13 mm, there is no convection, and heat transfer through the air is by conduction. But as the spacing is increased further, convection currents appear in the air space, and the increase in heat transfer coefficient offsets any benefit obtained by the thicker air layer. As a result, the heat transfer coefficient remains nearly constant, as shown in Figure 9-37. Therefore, it makes no sense to use an air space thicker than 13 mm in a double-pane window unless a thin polyester film is used to divide the air space into two halves to suppress convection currents. The film provides added insulation without adding much to the weight or cost of the double-pane window. The thermal resistance of the window can be increased further by using triple- or quadruple-pane windows whenever it is economical to do so. Note that using a triple-pane window instead of a double-pane reduces the rate of heat transfer through the center section of the window by about one-third.

Another way of reducing conduction heat transfer through a double-pane window is to use a *less-conducting fluid* such as argon or krypton to fill the gap between the glasses instead of air. The gap in this case needs to be well sealed to prevent the gas from leaking outside. Of course, another alternative is to evacuate the gap between the glasses completely, but it is not practical to do so.

Edge-of-Glass U -Factor of a Window

The glasses in double- and triple-pane windows are kept apart from each other at a uniform distance by **spacers** made of metals or insulators like aluminum, fiberglass, wood, and butyl. Continuous spacer strips are placed around the glass perimeter to provide an edge seal as well as uniform spacing. However, the spacers also serve as undesirable “thermal bridges” between the glasses, which are at different temperatures, and this short-circuiting may increase heat transfer through the window considerably. Heat transfer in the edge region of a window is two-dimensional, and lab measurements indicate that the edge effects are limited to a 6.5-cm-wide band around the perimeter of the glass.

The U -factor for the edge region of a window is given in Figure 9–38 relative to the U -factor for the center region of the window. The curve would be a straight diagonal line if the two U -values were equal to each other. Note that this is almost the case for insulating spacers such as wood and fiberglass. But the U -factor for the edge region can be twice that of the center region for conducting spacers such as those made of aluminum. Values for steel spacers fall between the two curves for metallic and insulating spacers. The edge effect is not applicable to single-pane windows.

Frame U -Factor

The framing of a window consists of the entire window except the glazing. Heat transfer through the framing is difficult to determine because of the different window configurations, different sizes, different constructions, and different combination of materials used in the frame construction. The type of glazing such as single pane, double pane, and triple pane affects the thickness of the framing and thus heat transfer through the frame. Most frames are made of *wood, aluminum, vinyl, or fiberglass*. However, using a combination of these materials (such as aluminum-clad wood and vinyl-clad aluminum) is also common to improve appearance and durability.

Aluminum is a popular framing material because it is inexpensive, durable, and easy to manufacture, and does not rot or absorb water like wood. However, from a heat transfer point of view, it is the least desirable framing material because of its high thermal conductivity. It will come as no surprise that the U -factor of solid aluminum frames is the highest, and thus a window with aluminum framing will lose much more heat than a comparable window with wood or vinyl framing. Heat transfer through the aluminum framing members can be reduced by using plastic inserts between components to serve as thermal barriers. The thickness of these inserts greatly affects heat transfer through the frame. For aluminum frames without the plastic strips, the primary resistance to heat transfer is due to the interior surface heat transfer coefficient. The U -factors for various

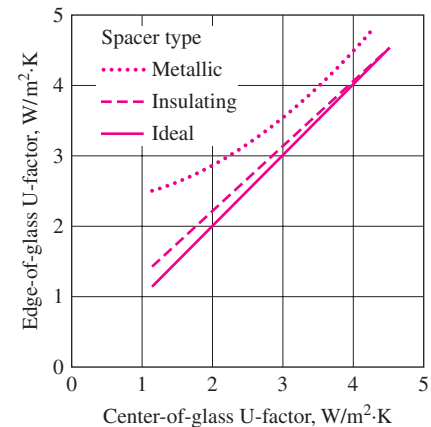


FIGURE 9–38

The edge-of-glass U -factor relative to the center-of-glass U -factor for windows with various spacers (from ASHRAE *Handbook of Fundamentals*, Ref. 1, Chap. 27, Fig. 2).

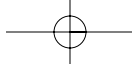


TABLE 9-4

Representative frame U -factors for fixed vertical windows (from ASHRAE *Handbook of Fundamentals*, Ref. 1, Chap. 27, Table 2)

Frame material	U -factor, $W/m^2 \cdot ^\circ C^*$
Aluminum:	
Single glazing (3 mm)	10.1
Double glazing (18 mm)	10.1
Triple glazing (33 mm)	10.1
Wood or vinyl:	
Single glazing (3 mm)	2.9
Double glazing (18 mm)	2.8
Triple glazing (33 mm)	2.7

*Multiply by 0.176 to convert to $Btu/h \cdot ft^2 \cdot ^\circ F$

TABLE 9-5

Combined convection and radiation heat transfer coefficient h_i at the inner surface of a vertical glass under still air conditions (in $W/m^2 \cdot ^\circ C^*$)

T_i , $^\circ C$	T_g , $^\circ C$	Glass emissivity, ϵ_g		
		0.05	0.20	0.84
20	17	2.6	3.5	7.1
20	15	2.9	3.8	7.3
20	10	3.4	4.2	7.7
20	5	3.7	4.5	7.9
20	0	4.0	4.8	8.1
20	-5	4.2	5.0	8.2
20	-10	4.4	5.1	8.3

*Multiply by 0.176 to convert to $Btu/h \cdot ft^2 \cdot ^\circ F$.

frames are listed in Table 9-4 as a function of spacer materials and the glazing unit thicknesses. Note that the U -factor of metal framing and thus the rate of heat transfer through a metal window frame is more than three times that of a wood or vinyl window frame.

Interior and Exterior Surface Heat Transfer Coefficients

Heat transfer through a window is also affected by the convection and radiation heat transfer coefficients between the glass surfaces and surroundings. The effects of convection and radiation on the inner and outer surfaces of glazings are usually combined into the combined convection and radiation heat transfer coefficients h_i and h_o , respectively. Under still air conditions, the combined heat transfer coefficient at the inner surface of a vertical window can be determined from

$$h_i = h_{conv} + h_{rad} = 1.77(T_g - T_i)^{0.25} + \frac{\epsilon_g \sigma (T_g^4 - T_i^4)}{T_g - T_i} \quad (W/m^2 \cdot ^\circ C) \quad (9-72)$$

where T_g = glass temperature in K, T_i = indoor air temperature in K, ϵ_g = emissivity of the inner surface of the glass exposed to the room (taken to be 0.84 for uncoated glass), and $\sigma = 5.67 \times 10^{-8} W/m^2 \cdot K^4$ is the Stefan-Boltzmann constant. Here the temperature of the interior surfaces facing the window is assumed to be equal to the indoor air temperature. This assumption is reasonable when the window faces mostly interior walls, but it becomes questionable when the window is exposed to heated or cooled surfaces or to other windows. The commonly used value of h_i for peak load calculation is

$$h_i = 8.29 W/m^2 \cdot ^\circ C = 1.46 Btu/h \cdot ft^2 \cdot ^\circ F \quad (\text{winter and summer})$$

which corresponds to the winter design conditions of $T_i = 22^\circ C$ and $T_g = -7^\circ C$ for uncoated glass with $\epsilon_g = 0.84$. But the same value of h_i can also be used for summer design conditions as it corresponds to summer conditions of $T_i = 24^\circ C$ and $T_g = 32^\circ C$. The values of h_i for various temperatures and glass emissivities are given in Table 9-5. The commonly used values of h_o for peak load calculations are the same as those used for outer wall surfaces ($34.0 W/m^2 \cdot ^\circ C$ for winter and $22.7 W/m^2 \cdot ^\circ C$ for summer).

Overall U -Factor of Windows

The overall U -factors for various kinds of windows and skylights are evaluated using computer simulations and laboratory testing for winter design conditions; representative values are given in Table 9-6. Test data may provide more accurate information for specific products and should be preferred when available. However, the values listed in the table can be used to obtain satisfactory results under various conditions in the absence of product-specific data. The U -factor of a fenestration product that differs considerably from the ones in the table can be determined by (1) determining the fractions of the area that are frame, center-of-glass, and edge-of-glass (assuming a 65-mm-wide band around the perimeter of each glazing),

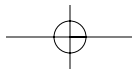


TABLE 9-6

Overall U -factors (heat transfer coefficients) for various windows and skylights in $W/m^2 \cdot ^\circ C$
(from ASHRAE *Handbook of Fundamentals*, Ref. 1, Chap. 27, Table 5)

Type →	Glass section (glazing) only			Aluminum frame (without thermal break)			Wood or vinyl frame					
	Center- of-glass	Edge-of- glass		Fixed	Double door	Sloped skylight	Fixed		Double door		Sloped skylight	
Frame width →	(Not applicable)			32 mm (1¼ in.)	53 mm (2 in.)	19 mm (¾ in.)	41 mm (1⅝ in.)		88 mm (3⅞ in.)		23 mm (⅞ in.)	
Spacer type →	—	Metal	Insul.	All	All	All	Metal	Insul.	Metal	Insul.	Metal	Insul.
Glazing Type												
Single Glazing												
3 mm (⅛ in.) glass	6.30	6.30	—	6.63	7.16	9.88	5.93	—	5.57	—	7.57	—
6.4 mm (¼ in.) acrylic	5.28	5.28	—	5.69	6.27	8.86	5.02	—	4.77	—	6.57	—
3 mm (⅛ in.) acrylic	5.79	5.79	—	6.16	6.71	9.94	5.48	—	5.17	—	7.63	—
Double Glazing (no coating)												
6.4 mm air space	3.24	3.71	3.34	3.90	4.55	6.70	3.26	3.16	3.20	3.09	4.37	4.22
12.7 mm air space	2.78	3.40	2.91	3.51	4.18	6.65	2.88	2.76	2.86	2.74	4.32	4.17
6.4 mm argon space	2.95	3.52	3.07	3.66	4.32	6.47	3.03	2.91	2.98	2.87	4.14	3.97
12.7 mm argon space	2.61	3.28	2.76	3.36	4.04	6.47	2.74	2.61	2.73	2.60	4.14	3.97
Double Glazing [$\epsilon = 0.1$, coating on one of the surfaces of air space (surface 2 or 3, counting from the outside toward inside)]												
6.4 mm air space	2.44	3.16	2.60	3.21	3.89	6.04	2.59	2.46	2.60	2.47	3.73	3.53
12.7 mm air space	1.82	2.71	2.06	2.67	3.37	6.04	2.06	1.92	2.13	1.99	3.73	3.53
6.4 mm argon space	1.99	2.83	2.21	2.82	3.52	5.62	2.21	2.07	2.26	2.12	3.32	3.09
12.7 mm argon space	1.53	2.49	1.83	2.42	3.14	5.71	1.82	1.67	1.91	1.78	3.41	3.19
Triple Glazing (no coating)												
6.4 mm air space	2.16	2.96	2.35	2.97	3.66	5.81	2.34	2.18	2.36	2.21	3.48	3.24
12.7 mm air space	1.76	2.67	2.02	2.62	3.33	5.67	2.01	1.84	2.07	1.91	3.34	3.09
6.4 mm argon space	1.93	2.79	2.16	2.77	3.47	5.57	2.15	1.99	2.19	2.04	3.25	3.00
12.7 mm argon space	1.65	2.58	1.92	2.52	3.23	5.53	1.91	1.74	1.98	1.82	3.20	2.95
Triple Glazing [$\epsilon = 0.1$, coating on one of the surfaces of air spaces (surfaces 3 and 5, counting from the outside toward inside)]												
6.4 mm air space	1.53	2.49	1.83	2.42	3.14	5.24	1.81	1.64	1.89	1.73	2.92	2.66
12.7 mm air space	0.97	2.05	1.38	1.92	2.66	5.10	1.33	1.15	1.46	1.30	2.78	2.52
6.4 mm argon space	1.19	2.23	1.56	2.12	2.85	4.90	1.52	1.35	1.64	1.47	2.59	2.33
12.7 mm argon space	0.80	1.92	1.25	1.77	2.51	4.86	1.18	1.01	1.33	1.17	2.55	2.28

Notes:

(1) Multiply by 0.176 to obtain U -factors in $Btu/h \cdot ft^2 \cdot ^\circ F$.

(2) The U -factors in this table include the effects of surface heat transfer coefficients and are based on winter conditions of $-18^\circ C$ outdoor air and $21^\circ C$ indoor air temperature, with 24 km/h (15 mph) winds outdoors and zero solar flux. Small changes in indoor and outdoor temperatures will not affect the overall U -factors much. Windows are assumed to be vertical, and the skylights are tilted 20° from the horizontal with upward heat flow. Insulation spacers are wood, fiberglass, or butyl. Edge-of-glass effects are assumed to extend the 65-mm band around perimeter of each glazing. The product sizes are 1.2 m \times 1.8 m for fixed windows, 1.8 m \times 2.0 m for double-door windows, and 1.2 m \times 0.6 m for the skylights, but the values given can also be used for products of similar sizes. All data are based on 3-mm (⅛-in.) glass unless noted otherwise.

(2) determining the U -factors for each section (the center-of-glass and edge-of-glass U -factors can be taken from the first two columns of Table 9–6 and the frame U -factor can be taken from Table 9–5 or other sources), and (3) multiplying the area fractions and the U -factors for each section and adding them up (or from Eq. 9-68 for U_{window}).

Glazed wall systems can be treated as fixed windows. Also, the data for double-door windows can be used for single-glass doors. Several observations can be made from the data in the table:

1. Skylight U -factors are considerably greater than those of vertical windows. This is because the skylight area, including the curb, can be 13 to 240 percent greater than the rough opening area. The slope of the skylight also has some effect.
2. The U -factor of multiple-glazed units can be reduced considerably by filling cavities with argon gas instead of dry air. The performance of CO_2 -filled units is similar to those filled with argon. The U -factor can be reduced even further by filling the glazing cavities with krypton gas.
3. Coating the glazing surfaces with low-e (low-emissivity) films reduces the U -factor significantly. For multiple-glazed units, it is adequate to coat one of the two surfaces facing each other.
4. The thicker the air space in multiple-glazed units, the lower the U -factor, for a thickness of up to 13 mm ($\frac{1}{2}$ in.) of air space. For a specified number of glazings, the window with thicker air layers will have a lower U -factor. For a specified overall thickness of glazing, the higher the number of glazings, the lower the U -factor. Therefore, a triple-pane window with air spaces of 6.4 mm (two such air spaces) will have a lower U -value than a double-pane window with an air space of 12.7 mm.
5. Wood or vinyl frame windows have a considerably lower U -value than comparable metal-frame windows. Therefore, wood or vinyl frame windows are called for in energy-efficient designs.

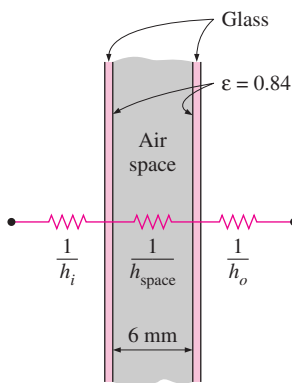


FIGURE 9–39
Schematic of Example 9–7.

EXAMPLE 9–7 U -Factor for Center-of-Glass Section of Windows

Determine the U -factor for the center-of-glass section of a double-pane window with a 6-mm air space for winter design conditions (Fig. 9–39). The glazings are made of clear glass that has an emissivity of 0.84. Take the average air space temperature at design conditions to be 0°C .

SOLUTION The U -factor for the center-of-glass section of a double-pane window is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer through the window is one-dimensional. 3 The thermal resistance of glass sheets is negligible.

Properties The emissivity of clear glass is 0.84.

Analysis Disregarding the thermal resistance of glass sheets, which are small, the U -factor for the center region of a double-pane window is determined from

$$\frac{1}{U_{\text{center}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

where h_i , h_{space} , and h_o are the heat transfer coefficients at the inner surface of the window, the air space between the glass layers, and the outer surface of the window, respectively. The values of h_i and h_o for winter design conditions were given earlier to be $h_i = 8.29 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_o = 34.0 \text{ W/m}^2 \cdot ^\circ\text{C}$. The effective emissivity of the air space of the double-pane window is

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} = 0.72$$

For this value of emissivity and an average air space temperature of 0°C , we read $h_{\text{space}} = 7.2 \text{ W/m}^2 \cdot ^\circ\text{C}$ from Table 9-3 for 6-mm-thick air space. Therefore,

$$\frac{1}{U_{\text{center}}} = \frac{1}{8.29} + \frac{1}{7.2} + \frac{1}{34.0} \rightarrow U_{\text{center}} = 3.46 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Discussion The center-of-glass U -factor value of $3.24 \text{ W/m}^2 \cdot ^\circ\text{C}$ in Table 9-6 (fourth row and second column) is obtained by using a standard value of $h_o = 29 \text{ W/m}^2 \cdot ^\circ\text{C}$ (instead of $34.0 \text{ W/m}^2 \cdot ^\circ\text{C}$) and $h_{\text{space}} = 6.5 \text{ W/m}^2 \cdot ^\circ\text{C}$ at an average air space temperature of -15°C .

EXAMPLE 9-8 Heat Loss through Aluminum Framed Windows

A fixed aluminum-framed window with glass glazing is being considered for an opening that is 4 ft high and 6 ft wide in the wall of a house that is maintained at 72°F (Fig. 9-40). Determine the rate of heat loss through the window and the inner surface temperature of the window glass facing the room when the outdoor air temperature is 15°F if the window is selected to be (a) $\frac{1}{8}$ -in. single glazing, (b) double glazing with an air space of $\frac{1}{2}$ in., and (c) low-e-coated triple glazing with an air space of $\frac{1}{2}$ in.

SOLUTION The rate of heat loss through an aluminum framed window and the inner surface temperature are to be determined from the cases of single-pane, double-pane, and low-e triple-pane windows.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer through the window is one-dimensional. 3 Thermal properties of the windows and the heat transfer coefficients are constant.

Properties The U -factors of the windows are given in Table 9-6.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively; U_{overall} is the U -factor (the overall heat transfer coefficient) of the window; and A_{window} is the window area, which is determined to be

$$A_{\text{window}} = \text{Height} \times \text{Width} = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2$$

The U -factors for the three cases can be determined directly from Table 9-6 to be 6.63, 3.51, and $1.92 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively, to be multiplied by the factor 0.176 to convert them to $\text{Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$. Also, the inner surface temperature of the window glass can be determined from Newton's law

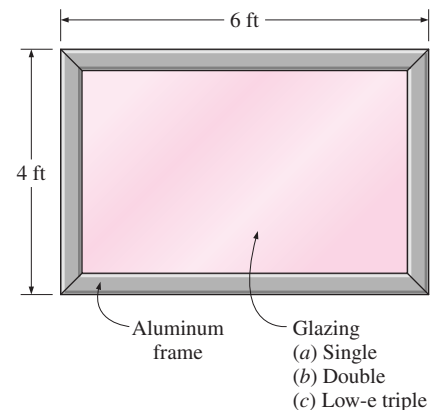


FIGURE 9-40

Schematic for Example 9-8.

$$\dot{Q}_{\text{window}} = h_i A_{\text{window}} (T_i - T_{\text{glass}}) \rightarrow T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}}$$

where h_i is the heat transfer coefficient on the inner surface of the window, which is determined from Table 9-5 to be $h_i = 8.3 \text{ W/m}^2 \cdot ^\circ\text{C} = 1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$. Then the rate of heat loss and the interior glass temperature for each case are determined as follows:

(a) Single glazing:

$$\dot{Q}_{\text{window}} = (6.63 \times 0.176 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)(72 - 15)^\circ\text{F} = \mathbf{1596 \text{ Btu/h}}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{1596 \text{ Btu/h}}{(1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)} = \mathbf{26.5^\circ\text{F}}$$

(b) Double glazing ($\frac{1}{2}$ in. air space):

$$\dot{Q}_{\text{window}} = (3.51 \times 0.176 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)(72 - 15)^\circ\text{F} = \mathbf{845 \text{ Btu/h}}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{845 \text{ Btu/h}}{(1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)} = \mathbf{47.9^\circ\text{F}}$$

(c) Triple glazing ($\frac{1}{2}$ in. air space, low-e coated):

$$\dot{Q}_{\text{window}} = (1.92 \times 0.176 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)(72 - 15)^\circ\text{F} = \mathbf{462 \text{ Btu/h}}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{462 \text{ Btu/h}}{(1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(24 \text{ ft}^2)} = \mathbf{58.8^\circ\text{F}}$$

Therefore, heat loss through the window will be reduced by 47 percent in the case of double glazing and by 71 percent in the case of triple glazing relative to the single-glazing case. Also, in the case of single glazing, the low inner-glass surface temperature will cause considerable discomfort in the occupants because of the excessive heat loss from the body by radiation. It is raised from 26.5°F , which is below freezing, to 47.9°F in the case of double glazing and to 58.8°F in the case of triple glazing.

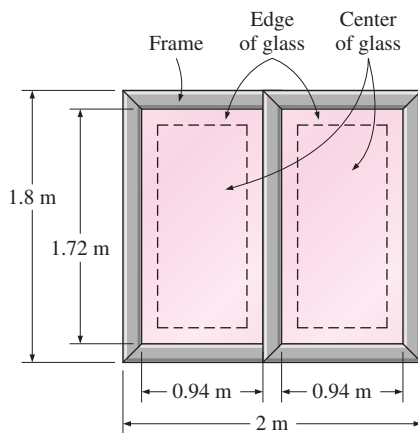


FIGURE 9-41
Schematic for Example 9-9.

EXAMPLE 9-9 U-Factor of a Double-Door Window

Determine the overall U -factor for a double-door-type, wood-framed double-pane window with metal spacers, and compare your result to the value listed in Table 9-6. The overall dimensions of the window are $1.80 \text{ m} \times 2.00 \text{ m}$, and the dimensions of each glazing are $1.72 \text{ m} \times 0.94 \text{ m}$ (Fig. 9-41).

SOLUTION The overall U -factor for a double-door type window is to be determined, and the result is to be compared to the tabulated value.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer through the window is one-dimensional.

Properties The U -factors for the various sections of windows are given in Tables 9-4 and 9-6.

Analysis The areas of the window, the glazing, and the frame are

$$A_{\text{window}} = \text{Height} \times \text{Width} = (1.8 \text{ m})(2.0 \text{ m}) = 3.60 \text{ m}^2$$

$$A_{\text{glazing}} = 2 \times (\text{Height} \times \text{Width}) = 2(1.72 \text{ m})(0.94 \text{ m}) = 3.23 \text{ m}^2$$

$$A_{\text{frame}} = A_{\text{window}} - A_{\text{glazing}} = 3.60 - 3.23 = 0.37 \text{ m}^2$$

The edge-of-glass region consists of a 6.5-cm-wide band around the perimeter of the glazings, and the areas of the center and edge sections of the glazing are determined to be

$$A_{\text{center}} = 2 \times (\text{Height} \times \text{Width}) = 2(1.72 - 0.13 \text{ m})(0.94 - 0.13 \text{ m}) = 2.58 \text{ m}^2$$

$$A_{\text{edge}} = A_{\text{glazing}} - A_{\text{center}} = 3.23 - 2.58 = 0.65 \text{ m}^2$$

The U -factor for the frame section is determined from Table 9-4 to be $U_{\text{frame}} = 2.8 \text{ W/m}^2 \cdot ^\circ\text{C}$. The U -factors for the center and edge sections are determined from Table 9-6 (fifth row, second and third columns) to be $U_{\text{center}} = 3.24 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $U_{\text{edge}} = 3.71 \text{ W/m}^2 \cdot ^\circ\text{C}$. Then the overall U -factor of the entire window becomes

$$\begin{aligned} U_{\text{window}} &= (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}} \\ &= (3.24 \times 2.58 + 3.71 \times 0.65 + 2.8 \times 0.37) / 3.60 \\ &= \mathbf{3.28 \text{ W/m}^2 \cdot ^\circ\text{C}} \end{aligned}$$

The overall U -factor listed in Table 9-6 for the specified type of window is $3.20 \text{ W/m}^2 \cdot ^\circ\text{C}$, which is sufficiently close to the value obtained above.

SUMMARY

In this chapter, we have considered *natural convection* heat transfer where any fluid motion occurs by natural means such as buoyancy. The *volume expansion coefficient* of a substance represents the variation of the density of that substance with temperature at constant pressure, and for an ideal gas, it is expressed as $\beta = 1/T$, where T is the absolute temperature in K or R.

The flow regime in natural convection is governed by a dimensionless number called the *Grashof number*, which represents the ratio of the buoyancy force to the viscous force acting on the fluid and is expressed as

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where L_c is the *characteristic length*, which is the height L for a vertical plate and the diameter D for a horizontal cylinder. The correlations for the Nusselt number $\text{Nu} = hL_c/k$ in natural convection are expressed in terms of the *Rayleigh number* defined as

$$\text{Ra}_L = \text{Gr}_L \text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \text{Pr}$$

Nusselt number relations for various surfaces are given in Table 9-1. All fluid properties are evaluated at the film tempera-

ture of $T_f = \frac{1}{2}(T_s + T_\infty)$. The outer surface of a vertical cylinder can be treated as a vertical plate when the curvature effects are negligible. The characteristic length for a horizontal surface is $L_c = A_s/p$, where A_s is the surface area and p is the perimeter.

The average Nusselt number for vertical isothermal *parallel plates* of spacing S and height L is given as

$$\text{Nu} = \frac{hS}{k} = \left[\frac{576}{(\text{Ra}_S S/L)^2} + \frac{2.873}{(\text{Ra}_S S/L)^{0.5}} \right]^{-0.5}$$

The optimum fin spacing for a vertical heat sink and the Nusselt number for optimally spaced fins is

$$S_{\text{opt}} = 2.714 \left(\frac{S^3 L}{\text{Ra}_S} \right)^{0.25} = 2.714 \frac{L}{\text{Ra}_L^{0.25}} \text{ and } \text{Nu} = \frac{hS_{\text{opt}}}{k} = 1.307$$

In a *horizontal rectangular enclosure* with the hotter plate at the top, heat transfer is by pure conduction and $\text{Nu} = 1$. When the hotter plate is at the bottom, the Nusselt is

$$\text{Nu} = 1 + 1.44 \left[1 - \frac{1708}{\text{Ra}_L} \right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1 \right]^+ \quad \text{Ra}_L < 10^8$$

The notation $[]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. For *vertical horizontal enclosures*, the Nusselt number can be determined from

$$\text{Nu} = 0.18 \left(\frac{\text{Pr}}{0.2 + \text{Pr}} \text{Ra}_L \right)^{0.29} \quad \begin{array}{l} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \text{Ra}_L \text{Pr}/(0.2 + \text{Pr}) > 10^3 \end{array}$$

$$\text{Nu} = 0.22 \left(\frac{\text{Pr}}{0.2 + \text{Pr}} \text{Ra}_L \right)^{0.28} \left(\frac{H}{L} \right)^{-1/4} \quad \begin{array}{l} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \text{Ra}_L < 10^{10} \end{array}$$

For aspect ratios greater than 10, Eqs. 9-54 and 9-55 should be used. For inclined enclosures, Eqs. 9-48 through 9-51 should be used.

For *concentric horizontal cylinders*, the rate of heat transfer through the annular space between the cylinders by natural convection per unit length is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)} (T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.386 \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{cyl}} \text{Ra}_L)^{1/4}$$

and

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3 (D_i^{-3/5} + D_o^{-3/5})^5}$$

For a *spherical enclosure*, the rate of heat transfer through the space between the spheres by natural convection is expressed as

$$\dot{Q} = k_{\text{eff}} \pi \left(\frac{D_i D_o}{L_c} \right) (T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.74 \left(\frac{\text{Pr}}{0.861 + \text{Pr}} \right)^{1/4} (F_{\text{sph}} \text{Ra}_L)^{1/4}$$

$$L_c = (D_o - D_i)/2$$

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$$

The quantity $k\text{Nu}$ is called the *effective thermal conductivity* of the enclosure, since a fluid in an enclosure behaves like a quiescent fluid whose thermal conductivity is $k\text{Nu}$ as a result of convection currents. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

For a given fluid, the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. Natural convection is negligible when $\text{Gr}/\text{Re}^2 < 0.1$, forced convection is negligible when $\text{Gr}/\text{Re}^2 > 10$, and neither is negligible when $0.1 < \text{Gr}/\text{Re}^2 < 10$.

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PROBLEMS*



Physical Mechanism of Natural Convection

9-1C What is natural convection? How does it differ from forced convection? What force causes natural convection currents?

9-2C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

9-3C Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times.

Will the egg cool faster or slower when the spacecraft is in space instead of on the ground? Explain.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon  are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon  are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

9-4C What is buoyancy force? Compare the relative magnitudes of the buoyancy force acting on a body immersed in these mediums: (a) air, (b) water, (c) mercury, and (d) an evacuated chamber.

9-5C When will the hull of a ship sink in water deeper: when the ship is sailing in fresh water or in sea water? Why?

9-6C A person weighs himself on a waterproof spring scale placed at the bottom of a 1-m-deep swimming pool. Will the person weigh more or less in water? Why?

9-7C Consider two fluids, one with a large coefficient of volume expansion and the other with a small one. In what fluid will a hot surface initiate stronger natural convection currents? Why? Assume the viscosity of the fluids to be the same.

9-8C Consider a fluid whose volume does not change with temperature at constant pressure. What can you say about natural convection heat transfer in this medium?

9-9C What do the lines on an interferometer photograph represent? What do closely packed lines on the same photograph represent?

9-10C Physically, what does the Grashof number represent? How does the Grashof number differ from the Reynolds number?

9-11 Show that the volume expansion coefficient of an ideal gas is $\beta = 1/T$, where T is the absolute temperature.

Natural Convection over Surfaces

9-12C How does the Rayleigh number differ from the Grashof number?

9-13C Under what conditions can the outer surface of a vertical cylinder be treated as a vertical plate in natural convection calculations?

9-14C Will a hot horizontal plate whose back side is insulated cool faster or slower when its hot surface is facing down instead of up?

9-15C Consider laminar natural convection from a vertical hot plate. Will the heat flux be higher at the top or at the bottom of the plate? Why?

9-16 A 10-m-long section of a 6-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 22°C. If the temperature and the emissivity of the outer surface of the pipe are 65°C and 0.8, respectively, determine the rate of heat loss from the pipe by (a) natural convection and (b) radiation.

9-17 Consider a wall-mounted power transistor that dissipates 0.18 W of power in an environment at 35°C. The transistor is 0.45 cm long and has a diameter of 0.4 cm. The emissivity of the outer surface of the transistor is 0.1, and the average temperature of the surrounding surfaces is 25°C. Dis-

regarding any heat transfer from the base surface, determine the surface temperature of the transistor. Use air properties at 100°C. **Answer: 183°C**

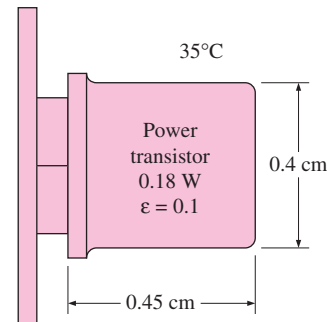




FIGURE P9-17

9-18  Reconsider Problem 9-17. Using EES (or other) software, investigate the effect of ambient temperature on the surface temperature of the transistor. Let the environment temperature vary from 10°C to 40°C and assume that the surrounding surfaces are 10°C colder than the environment temperature. Plot the surface temperature of the transistor versus the environment temperature, and discuss the results.

9-19E Consider a 2-ft × 2-ft thin square plate in a room at 75°F. One side of the plate is maintained at a temperature of 130°F, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.

9-20E  Reconsider Problem 9-19E. Using EES (or other) software, plot the rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature as the temperature varies from 80°F to 180°F, and discuss the results.

9-21 A 400-W cylindrical resistance heater is 1 m long and 0.5 cm in diameter. The resistance wire is placed horizontally in a fluid at 20°C. Determine the outer surface temperature of the resistance wire in steady operation if the fluid is (a) air and (b) water. Ignore any heat transfer by radiation. Use properties at 500°C for air and 40°C for water.

9-22 Water is boiling in a 12-cm-deep pan with an outer diameter of 25 cm that is placed on top of a stove. The ambient air and the surrounding surfaces are at a temperature of 25°C, and the emissivity of the outer surface of the pan is 0.95. Assuming the entire pan to be at an average temperature of 98°C, determine the rate of heat loss from the cylindrical side surface of the pan to the surroundings by (a) natural convection and (b) radiation. (c) If water is boiling at a rate of 2 kg/h at 100°C,

determine the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water. The heat of vaporization of water at 100°C is 2257 kJ/kg .

Answers: 46.2 W, 56.1 W, 0.082

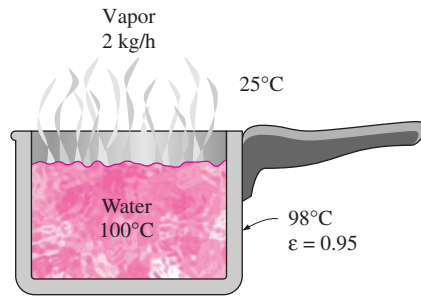


FIGURE P9-22

9-23 Repeat Problem 9-22 for a pan whose outer surface is polished and has an emissivity of 0.1.

9-24 In a plant that manufactures canned aerosol paints, the cans are temperature-tested in water baths at 55°C before they are shipped to ensure that they will withstand temperatures up to 55°C during transportation and shelving. The cans, moving on a conveyor, enter the open hot water bath, which is 0.5 m deep, 1 m wide, and 3.5 m long, and move slowly in the hot water toward the other end. Some of the cans fail the test and explode in the water bath. The water container is made of sheet metal, and the entire container is at about the same temperature as the hot water. The emissivity of the outer surface of the container is 0.7. If the temperature of the surrounding air and surfaces is 20°C , determine the rate of heat loss from the four side surfaces of the container (disregard the top surface, which is open).

The water is heated electrically by resistance heaters, and the cost of electricity is $\$0.085/\text{kWh}$. If the plant operates 24 h a day 365 days a year and thus 8760 h a year, determine the annual cost of the heat losses from the container for this facility.

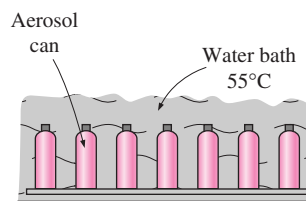


FIGURE P9-24

9-25 Reconsider Problem 9-24. In order to reduce the heating cost of the hot water, it is proposed to insulate the side and bottom surfaces of the container with 5-cm-thick fiberglass insulation ($k = 0.035\text{ W/m}\cdot^\circ\text{C}$) and to wrap the insulation with aluminum foil ($\varepsilon = 0.1$) in order to minimize the heat loss by

radiation. An estimate is obtained from a local insulation contractor, who proposes to do the insulation job for $\$350$, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves?

9-26 Consider a $15\text{-cm} \times 20\text{-cm}$ printed circuit board (PCB) that has electronic components on one side. The board is placed in a room at 20°C . The heat loss from the back surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down. Take the emissivity of the surface of the board to be 0.8 and assume the surrounding surfaces to be at the same temperature as the air in the room. *Answers: (a) 46.6°C , (b) 42.6°C , (c) 50.7°C*

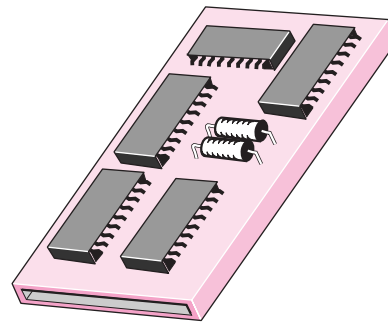




FIGURE P9-26

9-27  Reconsider Problem 9-26. Using EES (or other) software, investigate the effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board. Let the room temperature vary from 5°C to 35°C and the emissivity from 0.1 to 1.0. Plot the hot surface temperature for different orientations of the board as the functions of the room temperature and the emissivity, and discuss the results.

9-28  A manufacturer makes absorber plates that are $1.2\text{ m} \times 0.8\text{ m}$ in size for use in solar collectors. The back side of the plate is heavily insulated, while its front surface is coated with black chrome, which has an absorptivity of 0.87 for solar radiation and an emissivity of 0.09. Consider such a plate placed horizontally outdoors in calm air at 25°C . Solar radiation is incident on the plate at a rate of 700 W/m^2 . Taking the effective sky temperature to be 10°C , determine the equilibrium temperature of the absorber plate. What would your answer be if the absorber plate is made of ordinary aluminum plate that has a solar absorptivity of 0.28 and an emissivity of 0.07?

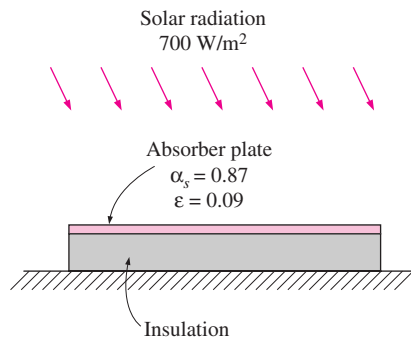
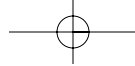


FIGURE P9-28

9-29 Repeat Problem 9-28 for an aluminum plate painted flat black (solar absorptivity 0.98 and emissivity 0.98) and also for a plate painted white (solar absorptivity 0.26 and emissivity 0.90).

9-30 The following experiment is conducted to determine the natural convection heat transfer coefficient for a horizontal cylinder that is 80 cm long and 2 cm in diameter. A 80-cm-long resistance heater is placed along the centerline of the cylinder, and the surfaces of the cylinder are polished to minimize the radiation effect. The two circular side surfaces of the cylinder are well insulated. The resistance heater is turned on, and the power dissipation is maintained constant at 40 W. If the average surface temperature of the cylinder is measured to be 120°C in the 20°C room air when steady operation is reached, determine the natural convection heat transfer coefficient. If the emissivity of the outer surface of the cylinder is 0.1 and a 5 percent error is acceptable, do you think we need to do any correction for the radiation effect? Assume the surrounding surfaces to be at 20°C also.

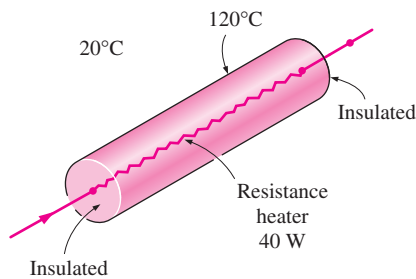


FIGURE P9-30

9-31 Thick fluids such as asphalt and waxes and the pipes in which they flow are often heated in order to reduce the viscosity of the fluids and thus to reduce the pumping costs. Consider the flow of such a fluid through a 100-m-long pipe of outer diameter 30 cm in calm ambient air at 0°C. The pipe is heated electrically, and a thermostat keeps the outer surface temperature of the pipe constant at 25°C. The emissivity of the outer surface of the pipe is 0.8, and the effective sky temperature is -30°C. Determine the power rating of the electric resistance heater, in kW, that needs to be used. Also, determine the cost of

electricity associated with heating the pipe during a 10-h period under the above conditions if the price of electricity is \$0.09/kWh. *Answers: 29.1 kW, \$26.2*

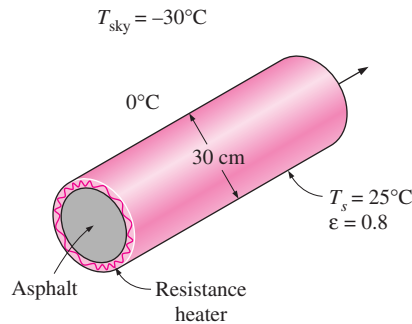


FIGURE P9-31

9-32 Reconsider Problem 9-31. To reduce the heating cost of the pipe, it is proposed to insulate it with sufficiently thick fiberglass insulation ($k = 0.035 \text{ W/m} \cdot ^\circ\text{C}$) wrapped with aluminum foil ($\epsilon = 0.1$) to cut down the heat losses by 85 percent. Assuming the pipe temperature to remain constant at 25°C, determine the thickness of the insulation that needs to be used. How much money will the insulation save during this 10-h period? *Answers: 1.3 cm, \$22.3*

9-33E Consider an industrial furnace that resembles a 13-ft-long horizontal cylindrical enclosure 8 ft in diameter whose end surfaces are well insulated. The furnace burns natural gas at a rate of 48 therms/h (1 therm = 100,000 Btu). The combustion efficiency of the furnace is 82 percent (i.e., 18 percent of the chemical energy of the fuel is lost through the flue gases as a result of incomplete combustion and the flue gases leaving the furnace at high temperature). If the heat loss from the outer surfaces of the furnace by natural convection and radiation is not to exceed 1 percent of the heat generated inside, determine the highest allowable surface temperature of the furnace. Assume the air and wall surface temperature of the room to be 75°F, and take the emissivity of the outer surface of the furnace to be 0.85. If the cost of natural gas is \$0.65/therm and the furnace operates 2800 h per year, determine the annual cost of this heat loss to the plant.

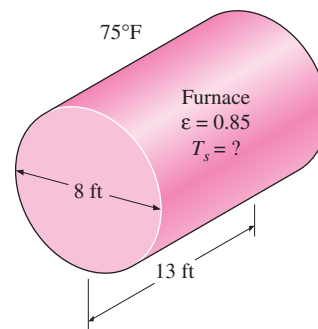
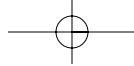


FIGURE P9-33



9-34 Consider a 1.2-m-high and 2-m-wide glass window with a thickness of 6 mm, thermal conductivity $k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$, and emissivity $\varepsilon = 0.9$. The room and the walls that face the window are maintained at 25°C , and the average temperature of the inner surface of the window is measured to be 5°C . If the temperature of the outdoors is -5°C , determine (a) the convection heat transfer coefficient on the inner surface of the window, (b) the rate of total heat transfer through the window, and (c) the combined natural convection and radiation heat transfer coefficient on the outer surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?

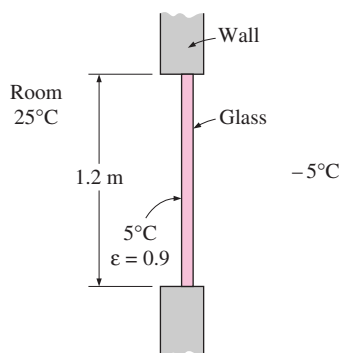


FIGURE P9-34

9-35 A 3-mm-diameter and 12-m-long electric wire is tightly wrapped with a 1.5-mm-thick plastic cover whose thermal conductivity and emissivity are $k = 0.15 \text{ W/m} \cdot ^\circ\text{C}$ and $\varepsilon = 0.9$. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to calm atmospheric air at $T_\infty = 30^\circ\text{C}$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Take the surrounding surfaces to be at about the same temperature as the air.

9-36 During a visit to a plastic sheeting plant, it was observed that a 60-m-long section of a 2-in. nominal (6.03-cm outer-diameter) steam pipe extended from one end of the plant to the other with no insulation on it. The temperature measurements at several locations revealed that the average temperature of the exposed surfaces of the steam pipe was 170°C , while the temperature of the surrounding air was 20°C . The outer surface of the pipe appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 20°C also, determine the rate of heat loss from the steam pipe.

Steam is generated in a gas furnace that has an efficiency of 78 percent, and the plant pays \$0.538 per therm (1 therm = 105,500 kJ) of natural gas. The plant operates 24 h a day 365 days a year, and thus 8760 h a year. Determine the annual cost of the heat losses from the steam pipe for this facility.

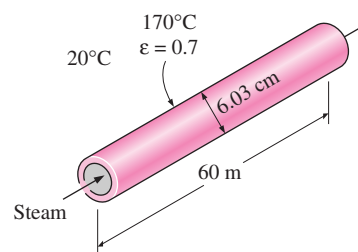



FIGURE P9-36

9-37  Reconsider Problem 9-36. Using EES (or other) software, investigate the effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss. Let the surface temperature vary from 100°C to 200°C . Plot the rate of heat loss and the annual cost as a function of the surface temperature, and discuss the results.

9-38 Reconsider Problem 9-36. In order to reduce heat losses, it is proposed to insulate the steam pipe with 5-cm-thick fiberglass insulation ($k = 0.038 \text{ W/m} \cdot ^\circ\text{C}$) and to wrap it with aluminum foil ($\varepsilon = 0.1$) in order to minimize the radiation losses. Also, an estimate is obtained from a local insulation contractor, who proposed to do the insulation job for \$750, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves? Assume the temperature of the steam pipe to remain constant at 170°C .

9-39 A 30-cm \times 30-cm circuit board that contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C . Each chip dissipates 0.05 W of power, and the emissivity of the chip surfaces is 0.7. Assuming the heat transfer from the back side of the circuit board to be negligible, and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the surface temperature of the chips. *Answer: 33.4°C*

9-40 Repeat Prob. 9-35 assuming the circuit board to be positioned horizontally with (a) chips facing up and (b) chips facing down.

9-41 The side surfaces of a 2-m-high cubic industrial furnace burning natural gas are not insulated, and the temperature at the outer surface of this section is measured to be 110°C . The temperature of the furnace room, including its surfaces, is 30°C , and the emissivity of the outer surface of the furnace is 0.7. It is proposed that this section of the furnace wall be insulated with glass wool insulation ($k = 0.038 \text{ W/m} \cdot ^\circ\text{C}$) wrapped by a reflective sheet ($\varepsilon = 0.2$) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 110°C , determine the thickness of the insulation that needs to be used.

The furnace operates continuously throughout the year and has an efficiency of 78 percent. The price of the natural gas is

\$0.55/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost \$550 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

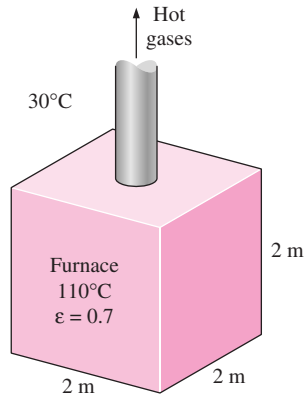


FIGURE P9-41

9-42 A 1.5-m-diameter, 5-m-long cylindrical propane tank is initially filled with liquid propane, whose density is 581 kg/m^3 . The tank is exposed to the ambient air at 25°C in calm weather. The outer surface of the tank is polished so that the radiation heat transfer is negligible. Now a crack develops at the top of the tank, and the pressure inside drops to 1 atm while the temperature drops to -42°C , which is the boiling temperature of propane at 1 atm. The heat of vaporization of propane at 1 atm is 425 kJ/kg . The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at -42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the tank to empty if it is not insulated.

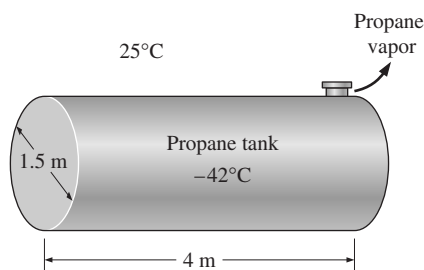


FIGURE P9-42

9-43E An average person generates heat at a rate of 287 Btu/h while resting in a room at 77°F . Assuming one-quarter of this heat is lost from the head and taking the emissivity of the skin to be 0.9, determine the average surface temperature of the head when it is not covered. The head can be approximated as a 12-in.-diameter sphere, and the interior surfaces of the room can be assumed to be at the room temperature.

9-44 An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 10 percent of the electrical energy it consumes into light while converting the remaining 90 percent into heat. The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation. Consider an 8-cm-diameter 60-W light bulb in a room at 25°C . The emissivity of the glass is 0.9. Assuming that 10 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself by natural convection and radiation, determine the equilibrium temperature of the glass bulb. Assume the interior surfaces of the room to be at room temperature. *Answer: 169°C*

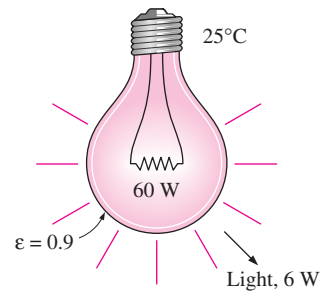



FIGURE P9-44

9-45 A 40-cm-diameter, 110-cm-high cylindrical hot water tank is located in the bathroom of a house maintained at 20°C . The surface temperature of the tank is measured to be 44°C and its emissivity is 0.4. Taking the surrounding surface temperature to be also 20°C , determine the rate of heat loss from all surfaces of the tank by natural convection and radiation.

9-46 A 28-cm-high, 18-cm-long, and 18-cm-wide rectangular container suspended in a room at 24°C is initially filled with cold water at 2°C . The surface temperature of the container is observed to be nearly the same as the water temperature inside. The emissivity of the container surface is 0.6, and the temperature of the surrounding surfaces is about the same as the air temperature. Determine the water temperature in the container after 3 h, and the average rate of heat transfer to the water. Assume the heat transfer coefficient on the top and bottom surfaces to be the same as that on the side surfaces.

9-47  Reconsider Problem 9-46. Using EES (or other) software, plot the water temperature in the container as a function of the heating time as the time varies from 30 min to 10 h, and discuss the results.

9-48 A room is to be heated by a coal-burning stove, which is a cylindrical cavity with an outer diameter of 32 cm and a height of 70 cm. The rate of heat loss from the room is estimated to be 1.2 kW when the air temperature in the room is maintained constant at 24°C . The emissivity of the stove surface is 0.85 and the average temperature of the surrounding

wall surfaces is 17°C . Determine the surface temperature of the stove. Neglect the transfer from the bottom surface and take the heat transfer coefficient at the top surface to be the same as that on the side surface.

The heating value of the coal is $30,000\text{ kJ/kg}$, and the combustion efficiency is 65 percent. Determine the amount of coal burned a day if the stove operates 14 h a day.

9-49 The water in a 40-L tank is to be heated from 15°C to 45°C by a 6-cm-diameter spherical heater whose surface temperature is maintained at 85°C . Determine how long the heater should be kept on.

Natural Convection from Finned Surfaces and PCBs

9-50C Why are finned surfaces frequently used in practice? Why are the finned surfaces referred to as heat sinks in the electronics industry?

9-51C Why are heat sinks with closely packed fins not suitable for natural convection heat transfer, although they increase the heat transfer surface area more?

9-52C Consider a heat sink with optimum fin spacing. Explain how heat transfer from this heat sink will be affected by (a) removing some of the fins on the heat sink and (b) doubling the number of fins on the heat sink by reducing the fin spacing. The base area of the heat sink remains unchanged at all times.

9-53 Aluminum heat sinks of rectangular profile are commonly used to cool electronic components. Consider a 7.62-cm-long and 9.68-cm-wide commercially available heat sink whose cross section and dimensions are as shown in Figure P9-53. The heat sink is oriented vertically and is used to cool a power transistor that can dissipate up to 125 W of power. The back surface of the heat sink is insulated. The surfaces of the heat sink are untreated, and thus they have a low emissivity (under 0.1). Therefore, radiation heat transfer from the heat sink can be neglected. During an experiment conducted in room air at 22°C , the base temperature of the heat sink was measured to be 120°C when the power dissipation of the transistor was 15 W. Assuming the entire heat sink to be at the base temperature, determine the average natural convection heat transfer coefficient for this case. *Answer: $7.1\text{ W/m}^2 \cdot ^{\circ}\text{C}$*

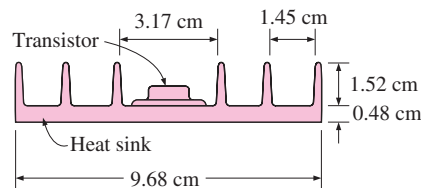


FIGURE P9-53

9-54 Reconsider the heat sink in Problem 9-53. In order to enhance heat transfer, a shroud (a thin rectangular metal plate) whose surface area is equal to the base area of the heat sink is placed very close to the tips of the fins such that the interfin

spaces are converted into rectangular channels. The base temperature of the heat sink in this case was measured to be 108°C . Noting that the shroud loses heat to the ambient air from both sides, determine the average natural convection heat transfer coefficient in this shrouded case. (For complete details, see Çengel and Zing, Ref. 9).

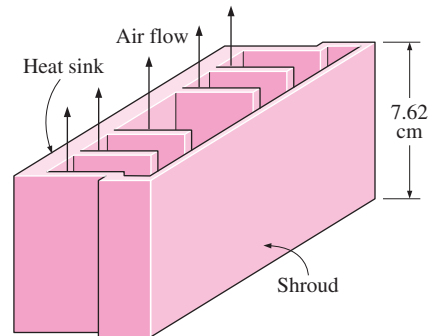



FIGURE P9-54

9-55E A 6-in.-wide and 8-in.-high vertical hot surface in 78°F air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.08 in. thick and 8 in. long in the vertical direction and have a height of 1.2 in. from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 180°F .

9-56E  Reconsider Problem 9-55E. Using EES (or other) software, investigate the effect of the length of the fins in the vertical direction on the optimum fin spacing and the rate of heat transfer by natural convection. Let the fin length vary from 2 in. to 10 in. Plot the optimum fin spacing and the rate of convection heat transfer as a function of the fin length, and discuss the results.

9-57 A 12.1-cm-wide and 18-cm-high vertical hot surface in 25°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction. Determine the optimum fin height and the rate of heat transfer by natural convection from the heat sink if the base temperature is 65°C .

Natural Convection inside Enclosures

9-58C The upper and lower compartments of a well-insulated container are separated by two parallel sheets of glass with an air space between them. One of the compartments is to be filled with a hot fluid and the other with a cold fluid. If it is desired that heat transfer between the two compartments be minimal, would you recommend putting the hot fluid into the upper or the lower compartment of the container? Why?

9-59C Someone claims that the air space in a double-pane window enhances the heat transfer from a house because of the natural convection currents that occur in the air space and

recommends that the double-pane window be replaced by a single sheet of glass whose thickness is equal to the sum of the thicknesses of the two glasses of the double-pane window to save energy. Do you agree with this claim?

9-60C Consider a double-pane window consisting of two glass sheets separated by a 1-cm-wide air space. Someone suggests inserting a thin vinyl sheet in the middle of the two glasses to form two 0.5-cm-wide compartments in the window in order to reduce natural convection heat transfer through the window. From a heat transfer point of view, would you be in favor of this idea to reduce heat losses through the window?

9-61C What does the effective conductivity of an enclosure represent? How is the ratio of the effective conductivity to thermal conductivity related to the Nusselt number?

9-62 Show that the thermal resistance of a rectangular enclosure can be expressed as $R = \delta/(Ak Nu)$, where k is the thermal conductivity of the fluid in the enclosure.

9-63E A vertical 4-ft-high and 6-ft-wide double-pane window consists of two sheets of glass separated by a 1-in. air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 65°F and 40°F, determine the rate of heat transfer through the window by (a) natural convection and (b) radiation. Also, determine the R -value of insulation of this window such that multiplying the inverse of the R -value by the surface area and the temperature difference gives the total rate of heat transfer through the window. The effective emissivity for use in radiation calculations between two large parallel glass plates can be taken to be 0.82.

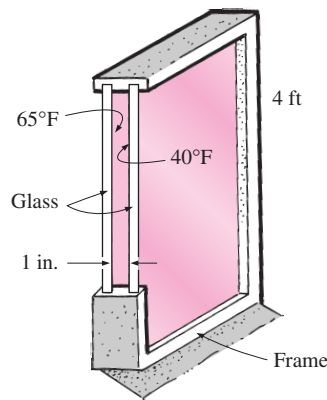


FIGURE P9-63E

9-64E Reconsider Problem 9-63E. Using EES (or other) software, investigate the effect of the air gap thickness on the rates of heat transfer by natural convection and radiation, and the R -value of insulation. Let the air gap thickness vary from 0.2 in. to 2.0 in. Plot the rates of heat transfer by natural convection and radiation, and the R -value of insulation as a function of the air gap thickness, and discuss the results.

9-65 Two concentric spheres of diameters 15 cm and 25 cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_1 = 350$ K and $T_2 = 275$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

9-66 Reconsider Problem 9-65. Using EES (or other) software, plot the rate of natural convection heat transfer as a function of the hot surface temperature of the sphere as the temperature varies from 300 K to 500 K, and discuss the results.

9-67 Flat-plate solar collectors are often tilted up toward the sun in order to intercept a greater amount of direct solar radiation. The tilt angle from the horizontal also affects the rate of heat loss from the collector. Consider a 2-m-high and 3-m-wide solar collector that is tilted at an angle θ from the horizontal. The back side of the absorber is heavily insulated. The absorber plate and the glass cover, which are spaced 2.5 cm from each other, are maintained at temperatures of 80°C and 40°C, respectively. Determine the rate of heat loss from the absorber plate by natural convection for $\theta = 0^\circ, 20^\circ,$ and 90° .

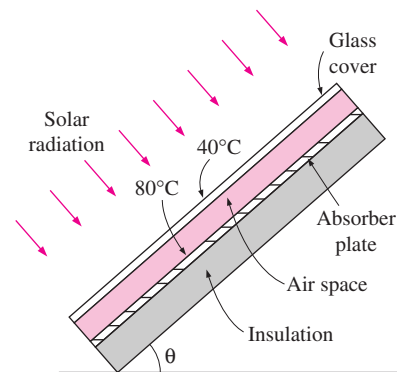


FIGURE P9-67

9-68 A simple solar collector is built by placing a 5-cm-diameter clear plastic tube around a garden hose whose outer diameter is 1.6 cm. The hose is painted black to maximize solar absorption, and some plastic rings are used to keep the spacing between the hose and the clear plastic cover constant. During a clear day, the temperature of the hose is measured to be 65°C,

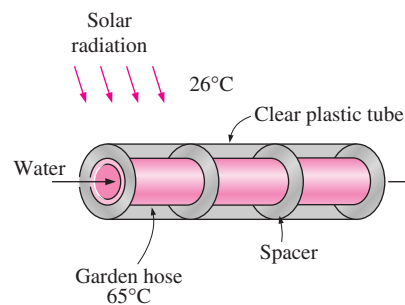



FIGURE P9-68

while the ambient air temperature is 26°C . Determine the rate of heat loss from the water in the hose per meter of its length by natural convection. Also, discuss how the performance of this solar collector can be improved. *Answer: 8.2 W*

9-69  Reconsider Problem 9-68. Using EES (or other) software, plot the rate of heat loss from the water by natural convection as a function of the ambient air temperature as the temperature varies from 4°C to 40°C , and discuss the results.

9-70 A vertical 1.3-m-high, 2.8-m-wide double-pane window consists of two layers of glass separated by a 2.2-cm air gap at atmospheric pressure. The room temperature is 26°C while the inner glass temperature is 18°C . Disregarding radiation heat transfer, determine the temperature of the outer glass layer and the rate of heat loss through the window by natural convection.

9-71 Consider two concentric horizontal cylinders of diameters 55 cm and 65 cm, and length 125 cm. The surfaces of the inner and outer cylinders are maintained at 46°C and 74°C , respectively. Determine the rate of heat transfer between the cylinders by natural convection if the annular space is filled with (a) water and (b) air.

Combined Natural and Forced Convection

9-72C When is natural convection negligible and when is it not negligible in forced convection heat transfer?

9-73C Under what conditions does natural convection enhance forced convection, and under what conditions does it hurt forced convection?

9-74C When neither natural nor forced convection is negligible, is it correct to calculate each independently and add them to determine the total convection heat transfer?

9-75 Consider a 5-m-long vertical plate at 85°C in air at 30°C . Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible.

Answer: 9.04 m/s

9-76 Reconsider Problem 9-75. Using EES (or other) software, plot the forced motion velocity above which natural convection heat transfer is negligible as a function of the plate temperature as the temperature varies from 50°C to 150°C , and discuss the results.

9-77 Consider a 5-m-long vertical plate at 60°C in water at 25°C . Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible. Take $\beta = 0.0004 \text{ K}^{-1}$ for water.

9-78 In a production facility, thin square plates $2 \text{ m} \times 2 \text{ m}$ in size coming out of the oven at 270°C are cooled by blowing ambient air at 30°C horizontally parallel to their surfaces. Determine the air velocity above which the natural convection effects on heat transfer are less than 10 percent and thus are negligible.

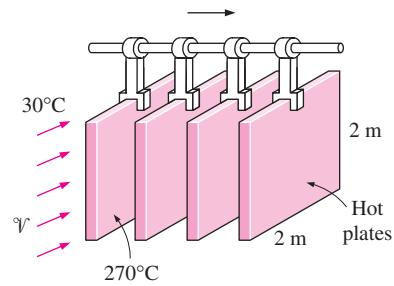


FIGURE P9-78

9-79 A 12-cm-high and 20-cm-wide circuit board houses 100 closely spaced logic chips on its surface, each dissipating 0.05 W. The board is cooled by a fan that blows air over the hot surface of the board at 35°C at a velocity of 0.5 m/s. The heat transfer from the back surface of the board is negligible. Determine the average temperature on the surface of the circuit board assuming the air flows vertically upwards along the 12-cm-long side by (a) ignoring natural convection and (b) considering the contribution of natural convection. Disregard any heat transfer by radiation.

Special Topic: Heat Transfer through Windows

9-80C Why are the windows considered in three regions when analyzing heat transfer through them? Name those regions and explain how the overall U -value of the window is determined when the heat transfer coefficients for all three regions are known.

9-81C Consider three similar double-pane windows with air gap widths of 5, 10, and 20 mm. For which case will the heat transfer through the window will be a minimum?

9-82C In an ordinary double-pane window, about half of the heat transfer is by radiation. Describe a practical way of reducing the radiation component of heat transfer.

9-83C Consider a double-pane window whose air space width is 20 mm. Now a thin polyester film is used to divide the air space into two 10-mm-wide layers. How will the film affect (a) convection and (b) radiation heat transfer through the window?

9-84C Consider a double-pane window whose air space is flashed and filled with argon gas. How will replacing the air in the gap by argon affect (a) convection and (b) radiation heat transfer through the window?

9-85C Is the heat transfer rate through the glazing of a double-pane window higher at the center or edge section of the glass area? Explain.

9-86C How do the relative magnitudes of U -factors of windows with aluminum, wood, and vinyl frames compare? Assume the windows are identical except for the frames.

9-87 Determine the U -factor for the center-of-glass section of a double-pane window with a 13-mm air space for winter

design conditions. The glazings are made of clear glass having an emissivity of 0.84. Take the average air space temperature at design conditions to be 10°C and the temperature difference across the air space to be 15°C .

9-88 A double-door wood-framed window with glass glazing and metal spacers is being considered for an opening that is 1.2 m high and 1.8 m wide in the wall of a house maintained at 20°C . Determine the rate of heat loss through the window and the inner surface temperature of the window glass facing the room when the outdoor air temperature is -8°C if the window is selected to be (a) 3-mm single glazing, (b) double glazing with an air space of 13 mm, and (c) low-e-coated triple glazing with an air space of 13 mm.

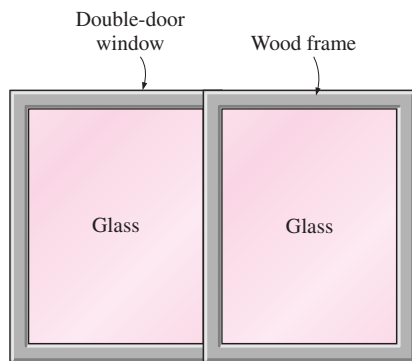


FIGURE P9-88

9-89 Determine the overall U -factor for a double-door-type wood-framed double-pane window with 13-mm air space and metal spacers, and compare your result to the value listed in Table 9-6. The overall dimensions of the window are $2.00\text{ m} \times 2.40\text{ m}$, and the dimensions of each glazing are $1.92\text{ m} \times 1.14\text{ m}$.

9-90 Consider a house in Atlanta, Georgia, that is maintained at 22°C and has a total of 20 m^2 of window area. The windows are double-door-type with wood frames and metal spacers. The glazing consists of two layers of glass with 12.7 mm of air space with one of the inner surfaces coated with reflective film. The winter average temperature of Atlanta is 11.3°C . Determine the average rate of heat loss through the windows in winter.

Answer: 456 W

9-91E Consider an ordinary house with R -13 walls (walls that have an R -value of $13\text{ h} \cdot \text{ft}^2 \cdot ^{\circ}\text{F}/\text{Btu}$). Compare this to the R -value of the common double-door windows that are double pane with $\frac{1}{4}$ in. of air space and have aluminum frames. If the windows occupy only 20 percent of the wall area, determine if more heat is lost through the windows or through the remaining 80 percent of the wall area. Disregard infiltration losses.

9-92 The overall U -factor of a fixed wood-framed window with double glazing is given by the manufacturer to be $U = 2.76\text{ W}/\text{m}^2 \cdot ^{\circ}\text{C}$ under the conditions of still air inside and

winds of 12 km/h outside. What will the U -factor be when the wind velocity outside is doubled? *Answer: $2.88\text{ W}/\text{m}^2 \cdot ^{\circ}\text{C}$*

9-93 The owner of an older house in Wichita, Kansas, is considering replacing the existing double-door type wood-framed single-pane windows with vinyl-framed double-pane windows with an air space of 6.4 mm. The new windows are of double-door type with metal spacers. The house is maintained at 22°C at all times, but heating is needed only when the outdoor temperature drops below 18°C because of the internal heat gain from people, lights, appliances, and the sun. The average winter temperature of Wichita is 7.1°C , and the house is heated by electric resistance heaters. If the unit cost of electricity is $\$0.07/\text{kWh}$ and the total window area of the house is 12 m^2 , determine how much money the new windows will save the home owner per month in winter.

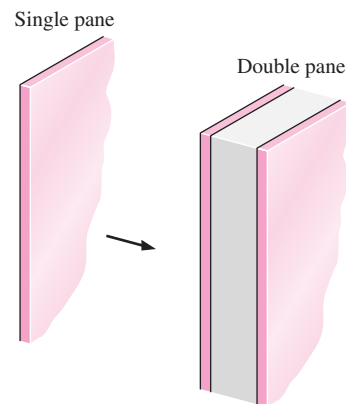


FIGURE P9-93

Review Problems

9-94E A 0.1-W small cylindrical resistor mounted on a lower part of a vertical circuit board is 0.3 in. long and has a diameter of 0.2 in. The view of the resistor is largely blocked by another circuit board facing it, and the heat transfer through the connecting wires is negligible. The air is free to flow through the large parallel flow passages between the boards as a result of natural convection currents. If the air temperature at the vicinity of the resistor is 120°F , determine the approximate surface temperature of the resistor. *Answer: 212°F*

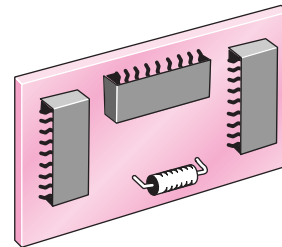


FIGURE P9-94E

9-95 An ice chest whose outer dimensions are 30 cm \times 40 cm \times 40 cm is made of 3-cm-thick styrofoam ($k = 0.033$ W/m \cdot $^{\circ}$ C). Initially, the chest is filled with 30 kg of ice at 0 $^{\circ}$ C, and the inner surface temperature of the ice chest can be taken to be 0 $^{\circ}$ C at all times. The heat of fusion of water at 0 $^{\circ}$ C is 333.7 kJ/kg, and the surrounding ambient air is at 20 $^{\circ}$ C. Disregarding any heat transfer from the 40 cm \times 40 cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely if the ice chest is subjected to (a) calm air and (b) winds at 50 km/h. Assume the heat transfer coefficient on the front, back, and top surfaces to be the same as that on the side surfaces.

9-96 An electronic box that consumes 180 W of power is cooled by a fan blowing air into the box enclosure. The dimensions of the electronic box are 15 cm \times 50 cm \times 50 cm, and all surfaces of the box are exposed to the ambient except the base surface. Temperature measurements indicate that the box is at an average temperature of 32 $^{\circ}$ C when the ambient temperature and the temperature of the surrounding walls are 25 $^{\circ}$ C. If the emissivity of the outer surface of the box is 0.85, determine the fraction of the heat lost from the outer surfaces of the electronic box.

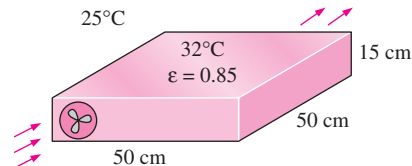


FIGURE P9-96

9-97 A 6-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel ($k = 15$ W/m \cdot $^{\circ}$ C) is used to store iced water at 0 $^{\circ}$ C in a room at 20 $^{\circ}$ C. The walls of the room are also at 20 $^{\circ}$ C. The outer surface of the tank is black (emissivity $\epsilon = 1$), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. Assuming the entire steel tank to be at 0 $^{\circ}$ C and thus the thermal resistance of the tank to be negligible, determine (a) the rate of heat transfer to the iced water in the tank and (b) the amount of ice at 0 $^{\circ}$ C that melts during a 24-h period.

Answers: (a) 15.4 kW, (b) 3988 kg

9-98 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m \cdot $^{\circ}$ C) separated by a 3-cm-wide air space. Determine the steady rate of heat transfer through this window and the temperature of its inner surface for a day during which the room is maintained at 20 $^{\circ}$ C while the temperature of the outdoors is 0 $^{\circ}$ C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m 2 \cdot $^{\circ}$ C and $h_2 = 25$ W/m 2 \cdot $^{\circ}$ C and disregard any heat transfer by radiation.

9-99 An electric resistance space heater is designed such that it resembles a rectangular box 50 cm high, 80 cm long, and

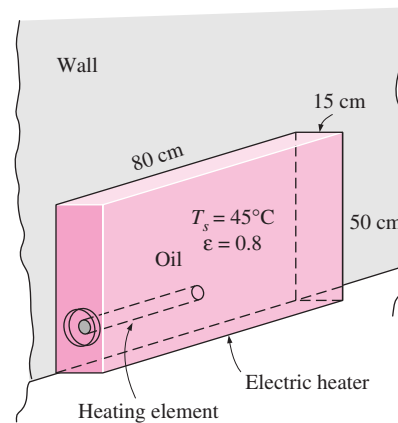


FIGURE P9-99

15 cm wide filled with 45 kg of oil. The heater is to be placed against a wall, and thus heat transfer from its back surface is negligible for safety considerations. The surface temperature of the heater is not to exceed 45 $^{\circ}$ C in a room at 25 $^{\circ}$ C. Disregarding heat transfer from the bottom and top surfaces of the heater in anticipation that the top surface will be used as a shelf, determine the power rating of the heater in W. Take the emissivity of the outer surface of the heater to be 0.8 and the average temperature of the ceiling and wall surfaces to be the same as the room air temperature.

Also, determine how long it will take for the heater to reach steady operation when it is first turned on (i.e., for the oil temperature to rise from 25 $^{\circ}$ C to 45 $^{\circ}$ C). State your assumptions in the calculations.

9-100 Skylights or “roof windows” are commonly used in homes and manufacturing facilities since they let natural light in during day time and thus reduce the lighting costs. However, they offer little resistance to heat transfer, and large amounts of energy are lost through them in winter unless they are equipped with a motorized insulating cover that can be used in cold weather and at nights to reduce heat losses. Consider a 1-m-wide and 2.5-m-long horizontal skylight on the roof of a house that is kept at 20 $^{\circ}$ C. The glazing of the skylight is made of a single layer of 0.5-cm-thick

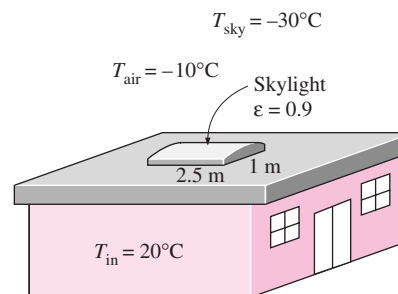


FIGURE P9-100

glass ($k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$ and $\varepsilon = 0.9$). Determine the rate of heat loss through the skylight when the air temperature outside is -10°C and the effective sky temperature is -30°C . Compare your result with the rate of heat loss through an equivalent surface area of the roof that has a common R-5.34 construction in SI units (i.e., a thickness-to-effective-thermal-conductivity ratio of $5.34 \text{ m}^2 \cdot ^\circ\text{C/W}$).

9-101 A solar collector consists of a horizontal copper tube of outer diameter 5 cm enclosed in a concentric thin glass tube of 9 cm diameter. Water is heated as it flows through the tube, and the annular space between the copper and glass tube is filled with air at 1 atm pressure. During a clear day, the temperatures of the tube surface and the glass cover are measured to be 60°C and 32°C , respectively. Determine the rate of heat loss from the collector by natural convection per meter length of the tube. *Answer: 17.4 W*

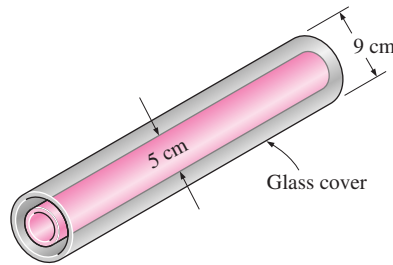


FIGURE P9-101

9-102 A solar collector consists of a horizontal aluminum tube of outer diameter 4 cm enclosed in a concentric thin glass tube of 7 cm diameter. Water is heated as it flows through the aluminum tube, and the annular space between the aluminum and glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 20 W per meter length, and the temperature of the ambient air outside is 30°C . Approximating the surfaces of the tube and the glass cover as being black (emissivity $\varepsilon = 1$) in radiation calculations and taking the effective sky temperature to be 20°C , determine the temperature of the aluminum tube when equilibrium is established (i.e., when the net heat loss from the tube by convection and radiation equals the amount of solar energy absorbed by the tube).

9-103E The components of an electronic system dissipating 180 W are located in a 4-ft-long horizontal duct whose cross-section is 6 in. \times 6 in. The components in the duct are cooled by forced air, which enters at 85°F at a rate of 22 cfm and leaves at 100°F . The surfaces of the sheet metal duct are not painted, and thus radiation heat transfer from the outer surfaces is negligible. If the ambient air temperature is 80°F , determine (a) the heat transfer from the outer surfaces of the duct to the ambient air by natural convection and (b) the average temperature of the duct.

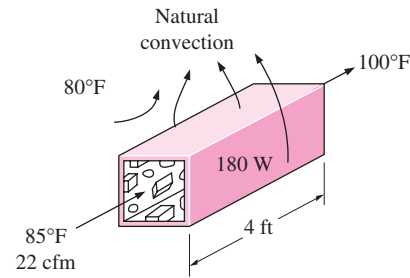


FIGURE P9-103E

9-104E Repeat Problem 9-103E for a circular horizontal duct of diameter 4 in.

9-105E Repeat Problem 9-103E assuming the fan fails and thus the entire heat generated inside the duct must be rejected to the ambient air by natural convection through the outer surfaces of the duct.

9-106 Consider a cold aluminum canned drink that is initially at a uniform temperature of 5°C . The can is 12.5 cm high and has a diameter of 6 cm. The emissivity of the outer surface of the can is 0.6. Disregarding any heat transfer from the bottom surface of the can, determine how long it will take for the average temperature of the drink to rise to 7°C if the surrounding air and surfaces are at 25°C . *Answer: 12.1 min*

9-107 Consider a 2-m-high electric hot water heater that has a diameter of 40 cm and maintains the hot water at 60°C . The tank is located in a small room at 20°C whose walls and the ceiling are at about the same temperature. The tank is placed in a 46-cm-diameter sheet metal shell of negligible thickness, and the space between the tank and the shell is filled with foam insulation. The average temperature and emissivity of the outer surface of the shell are 40°C and 0.7, respectively. The price of

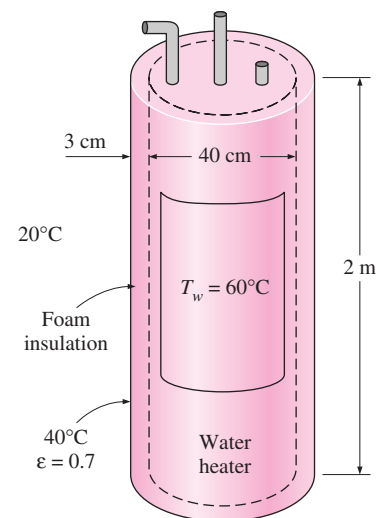


FIGURE P9-107

electricity is \$0.08/kWh. Hot water tank insulation kits large enough to wrap the entire tank are available on the market for about \$30. If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself? Disregard any heat loss from the top and bottom surfaces, and assume the insulation to reduce the heat losses by 80 percent.

9-108 During a plant visit, it was observed that a 1.5-m-high and 1-m-wide section of the vertical front section of a natural gas furnace wall was too hot to touch. The temperature measurements on the surface revealed that the average temperature of the exposed hot surface was 110°C , while the temperature of the surrounding air was 25°C . The surface appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 25°C also, determine the rate of heat loss from this furnace.

The furnace has an efficiency of 79 percent, and the plant pays \$0.75 per therm of natural gas. If the plant operates 10 h a day, 310 days a year, and thus 3100 h a year, determine the annual cost of the heat loss from this vertical hot surface on the front section of the furnace wall.

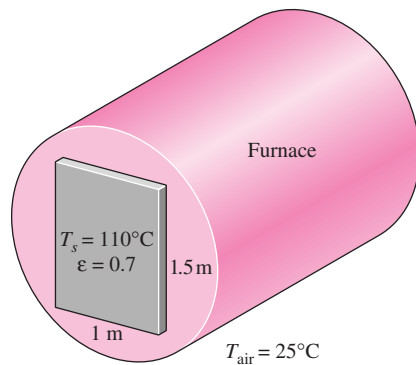


FIGURE P9-108

9-109 A group of 25 power transistors, dissipating 1.5 W each, are to be cooled by attaching them to a black-anodized square aluminum plate and mounting the plate on the wall of a room at 30°C . The emissivity of the transistor and the plate surfaces is 0.9. Assuming the heat transfer from the back side of the plate to be negligible and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the size of the plate if the average surface temperature of the plate is not to exceed 50°C . *Answer: 43 cm 3 43 cm*

9-110 Repeat Problem 9-109 assuming the plate to be positioned horizontally with (a) transistors facing up and (b) transistors facing down.

9-111E Hot water is flowing at an average velocity of 4 ft/s through a cast iron pipe ($k = 30 \text{ Btu/h} \cdot \text{ft} \cdot ^{\circ}\text{F}$) whose inner and outer diameters are 1.0 in. and 1.2 in., respectively. The pipe passes through a 50-ft-long section of a basement whose temperature is 60°F . The emissivity of the outer surface of the pipe

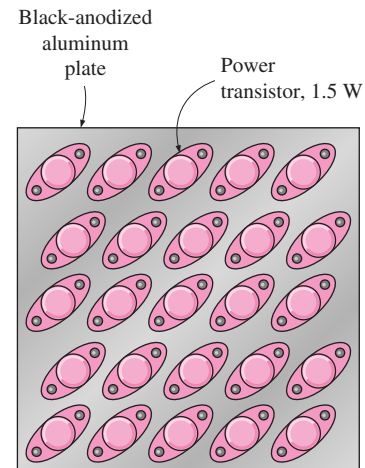


FIGURE P9-109

is 0.5, and the walls of the basement are also at about 60°F . If the inlet temperature of the water is 150°F and the heat transfer coefficient on the inner surface of the pipe is $30 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^{\circ}\text{F}$, determine the temperature drop of water as it passes through the basement.

9-112 Consider a flat-plate solar collector placed horizontally on the flat roof of a house. The collector is 1.5 m wide and 6 m long, and the average temperature of the exposed surface of the collector is 42°C . Determine the rate of heat loss from the collector by natural convection during a calm day when the ambient air temperature is 15°C . Also, determine the heat loss by radiation by taking the emissivity of the collector surface to be 0.9 and the effective sky temperature to be -30°C . *Answers: 1295 W, 2921 W*

9-113 Solar radiation is incident on the glass cover of a solar collector at a rate of 650 W/m^2 . The glass transmits 88 percent of the incident radiation and has an emissivity of 0.90. The hot water needs of a family in summer can be met completely by a

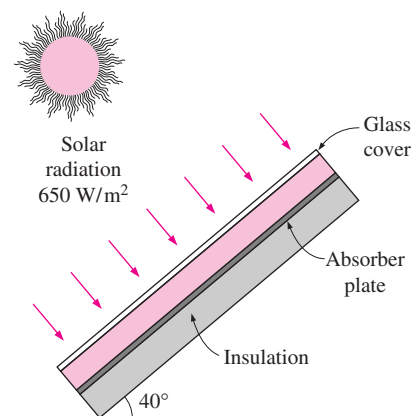


FIGURE P9-113

collector 1.5 m high and 2 m wide, and tilted 40° from the horizontal. The temperature of the glass cover is measured to be 40°C on a calm day when the surrounding air temperature is 20°C . The effective sky temperature for radiation exchange between the glass cover and the open sky is -40°C . Water enters the tubes attached to the absorber plate at a rate of 1 kg/min. Assuming the back surface of the absorber plate to be heavily insulated and the only heat loss occurs through the glass cover, determine (a) the total rate of heat loss from the collector, (b) the collector efficiency, which is the ratio of the amount of heat transferred to the water to the solar energy incident on the collector, and (c) the temperature rise of water as it flows through the collector.

Design and Essay Problems

9-114 Write a computer program to evaluate the variation of temperature with time of thin square metal plates that are removed from an oven at a specified temperature and placed vertically in a large room. The thickness, the size, the initial temperature, the emissivity, and the thermophysical properties of the plate as well as the room temperature are to be specified by the user. The program should evaluate the temperature of the plate at specified intervals and tabulate the results against time. The computer should list the assumptions made during calculations before printing the results.

For each step or time interval, assume the surface temperature to be constant and evaluate the heat loss during that time interval and the temperature drop of the plate as a result of this heat loss. This gives the temperature of the plate at the end of a time interval, which is to serve as the initial temperature of the plate for the beginning of the next time interval.

Try your program for 0.2-cm-thick vertical copper plates of $40\text{ cm} \times 40\text{ cm}$ in size initially at 300°C cooled in a room at 25°C . Take the surface emissivity to be 0.9. Use a time interval of 1 s in calculations, but print the results at 10-s intervals for a total cooling period of 15 min.

9-115 Write a computer program to optimize the spacing between the two glasses of a double-pane window. Assume the spacing is filled with dry air at atmospheric pressure. The program should evaluate the recommended practical value of the spacing to minimize the heat losses and list it when the size of the window (the height and the width) and the temperatures of the two glasses are specified.

9-116 Contact a manufacturer of aluminum heat sinks and obtain their product catalog for cooling electronic components by natural convection and radiation. Write an essay on how to select a suitable heat sink for an electronic component when its maximum power dissipation and maximum allowable surface temperature are specified.

9-117 The top surfaces of practically all flat-plate solar collectors are covered with glass in order to reduce the heat losses from the absorber plate underneath. Although the glass cover reflects or absorbs about 15 percent of the incident solar radiation, it saves much more from the potential heat losses from the absorber plate, and thus it is considered to be an essential part of a well-designed solar collector. Inspired by the energy efficiency of double-pane windows, someone proposes to use double glazing on solar collectors instead of a single glass. Investigate if this is a good idea for the town in which you live. Use local weather data and base your conclusion on heat transfer analysis and economic considerations.